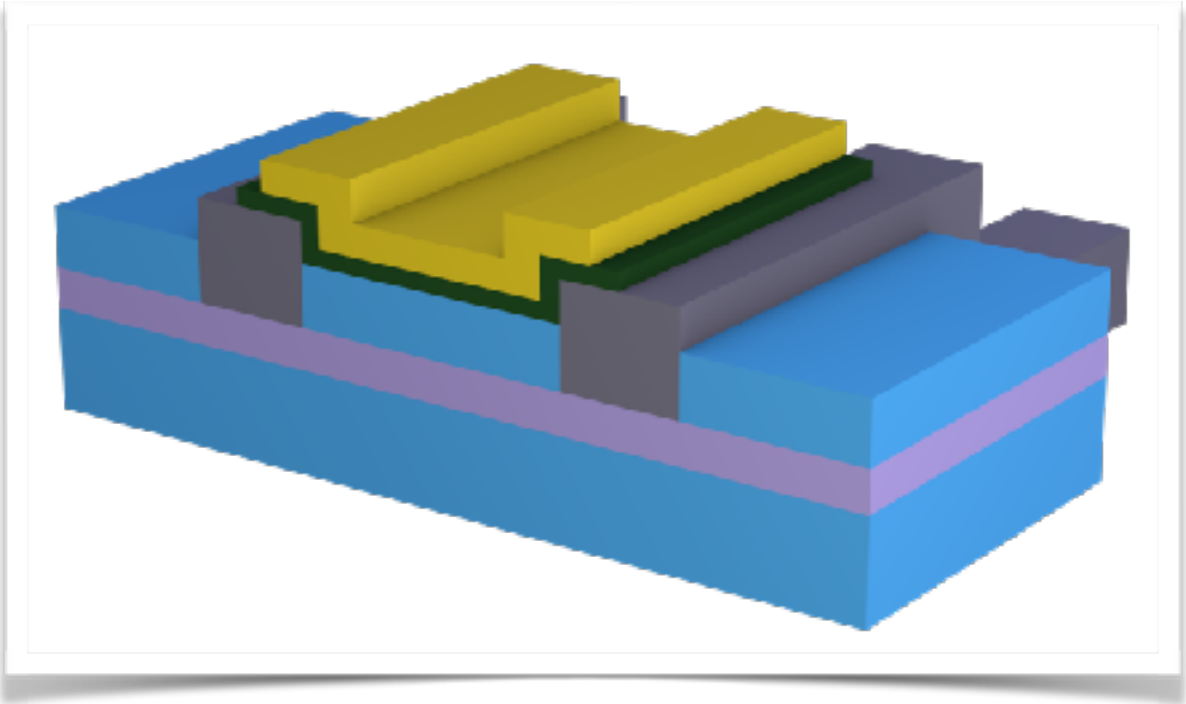
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Erwann Bocquillon

Topological insulators & topological superconductivity in HgTe heterostructures

Topological Matter School
25/08/2017



Physikalisches Institut (EP3)
Universität Würzburg, Am Hubland, D-97074 Würzburg
http://www.physik.uni-wuerzburg.de/EP3/

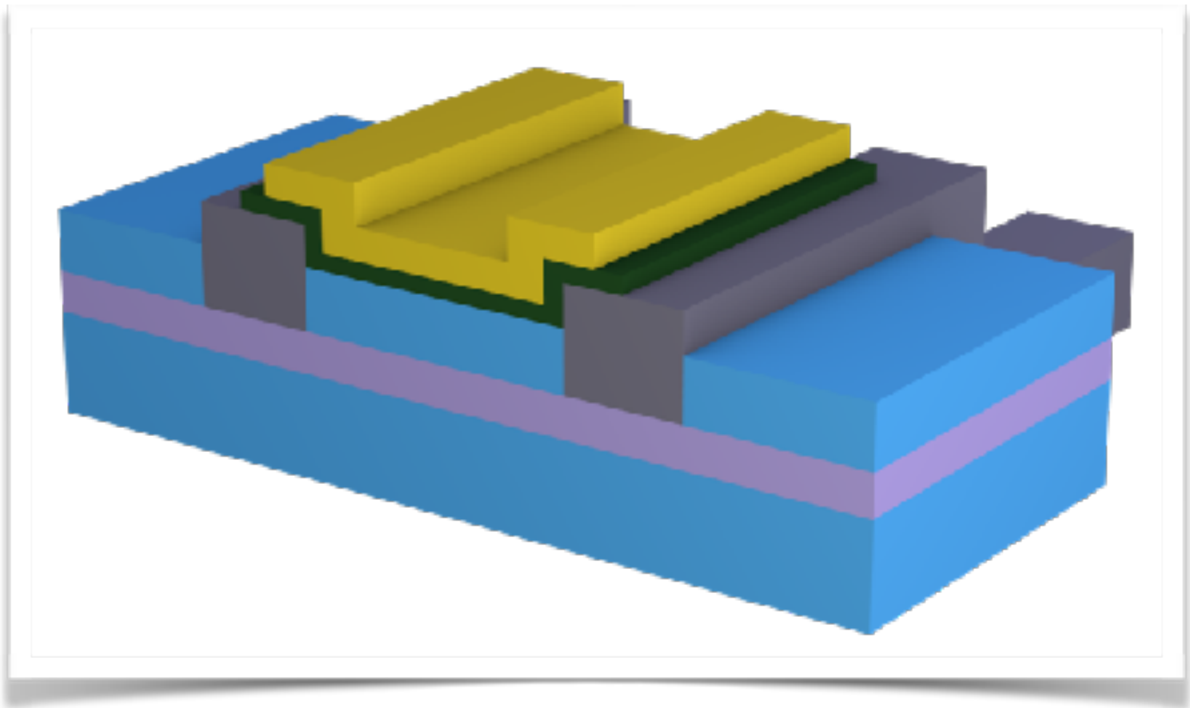
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

EP3

Erwann Bocquillon

Topological insulators & topological superconductivity in HgTe heterostructures

Topological Matter School
25/08/2017



Laboratoire Pierre Aigrain
Ecole Normale Supérieure
24 rue Lhomond, 75231 Paris Cedex 05 France
www.lpa.ens.fr

People involved

Uni. Würzburg

- **Students :** J. Wiedenmann, E. Liebhaber
  D. Mahler, A. Budewitz, K. Bendias,
  S. Hartinger
- **Staff :** C. Brüne
  H. Buhmann
  L.W. Molenkamp
- **Invited :** T.M. Klapwijk
- **Theory :** F. Domínguez, E.M. Hankiewicz

European Research Council
Established by the European Commission

Unterstützt von / Supported by
Alexander von Humboldt
Stiftung / Foundation

RIKEN, Tōkyō

- R.S. Deacon, K. Ishibashi, S. Tarucha

Outline

I. HgTe as a topological material

A. Basics of HgCdTe compounds

B. HgTe quantum wells

C. 3D layers & strain engineering

II. Search for topological superconductivity in HgTe

A. Foreword on topological superconductivity

B. Physics of a Josephson junction

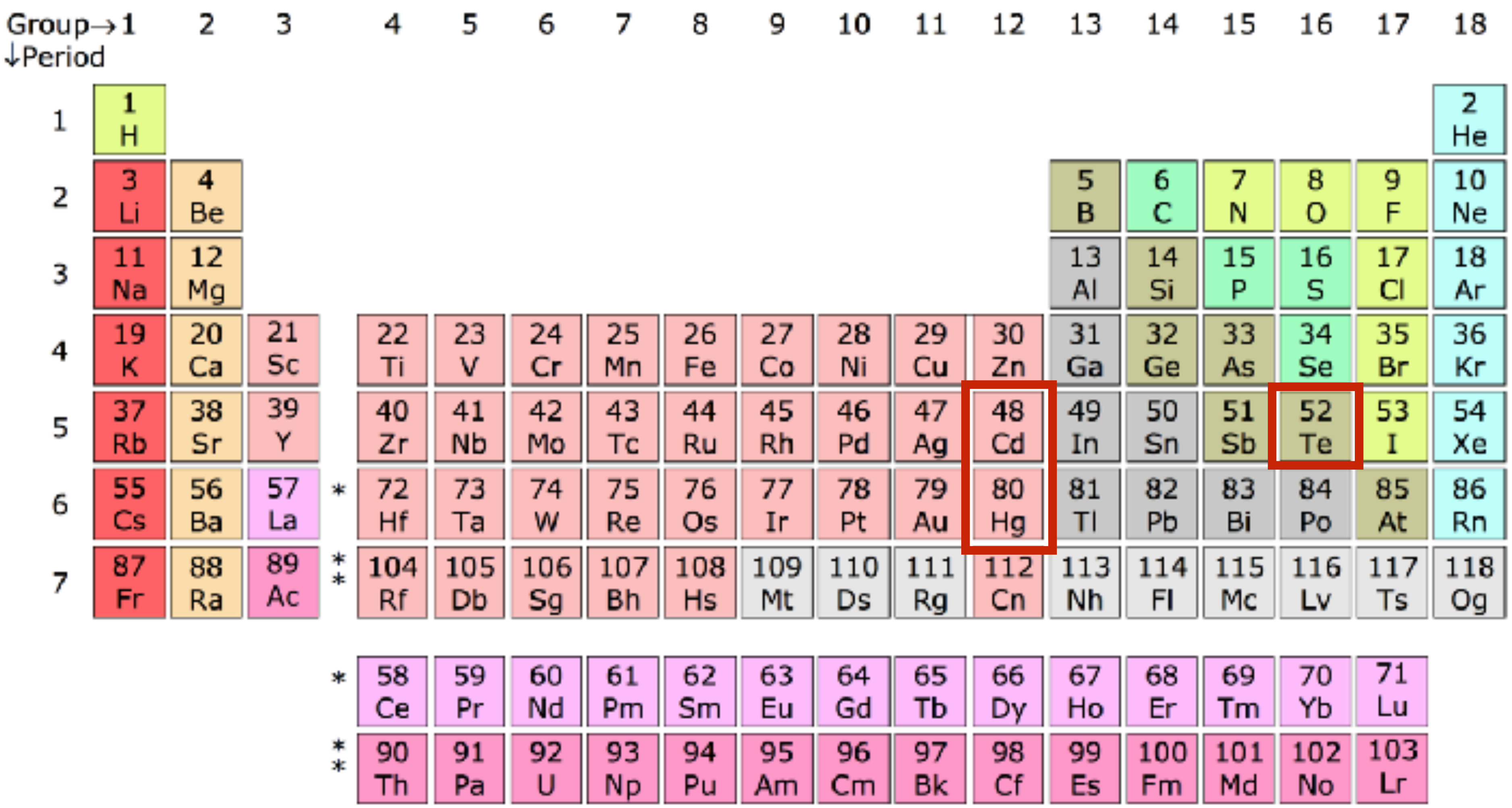
C. Search for gapless ABS in topological JJs

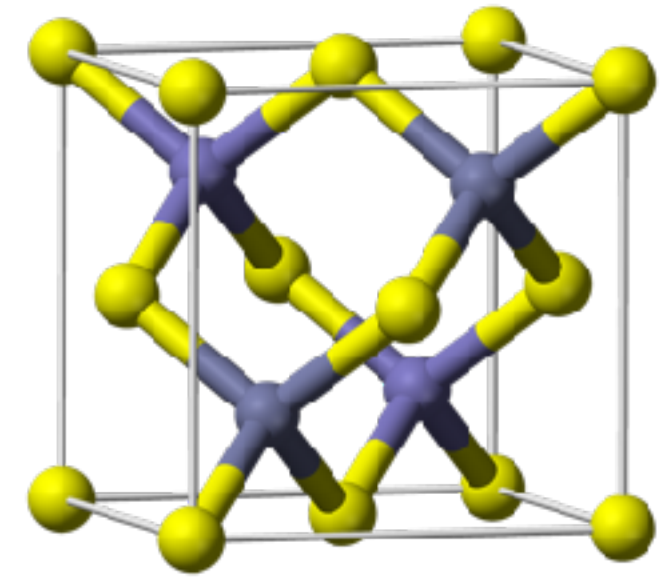
D. Induced superconductivity in S-N junctions

Part I - Topological phases in HgTe

A. Basics of HgCdTe compounds
B. HgTe quantum wells
C. 3D layers & strain engineering

I-A. Basics of HgTe

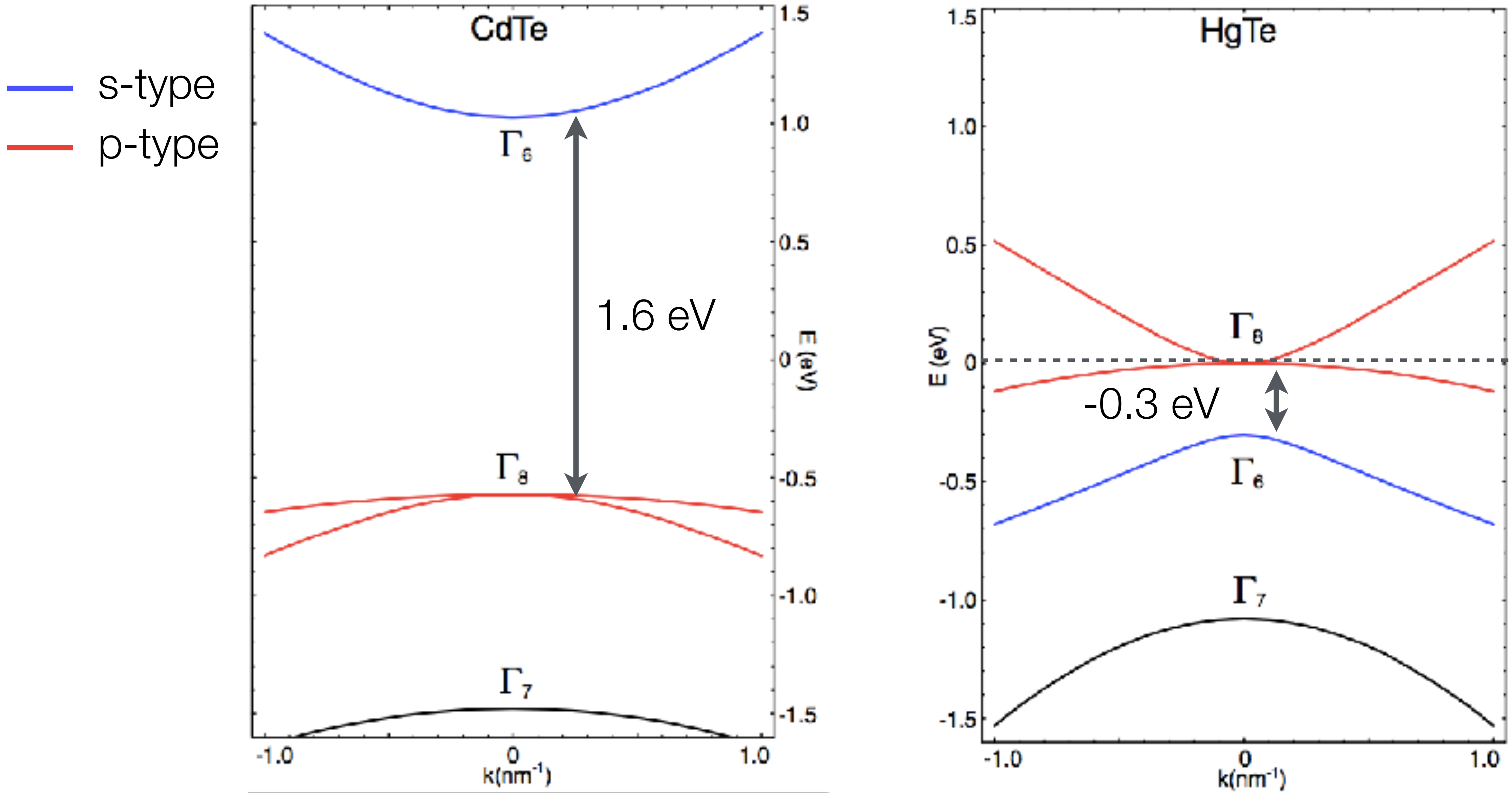




- ▹ II-VI semiconductors with ZnS structure (2 fcc with tetrahedral coordination)
- ▹ heavy + strong SOC ⇒ strong relativistic corrections
- ▹ different lattice constants (0.3%) ⇒ strain !

$$a_{\mathrm{HgTe}} = 0.646\,\mathrm{nm}$$
$$a_{\mathrm{CdTe}} = 0.648\,\mathrm{nm}$$

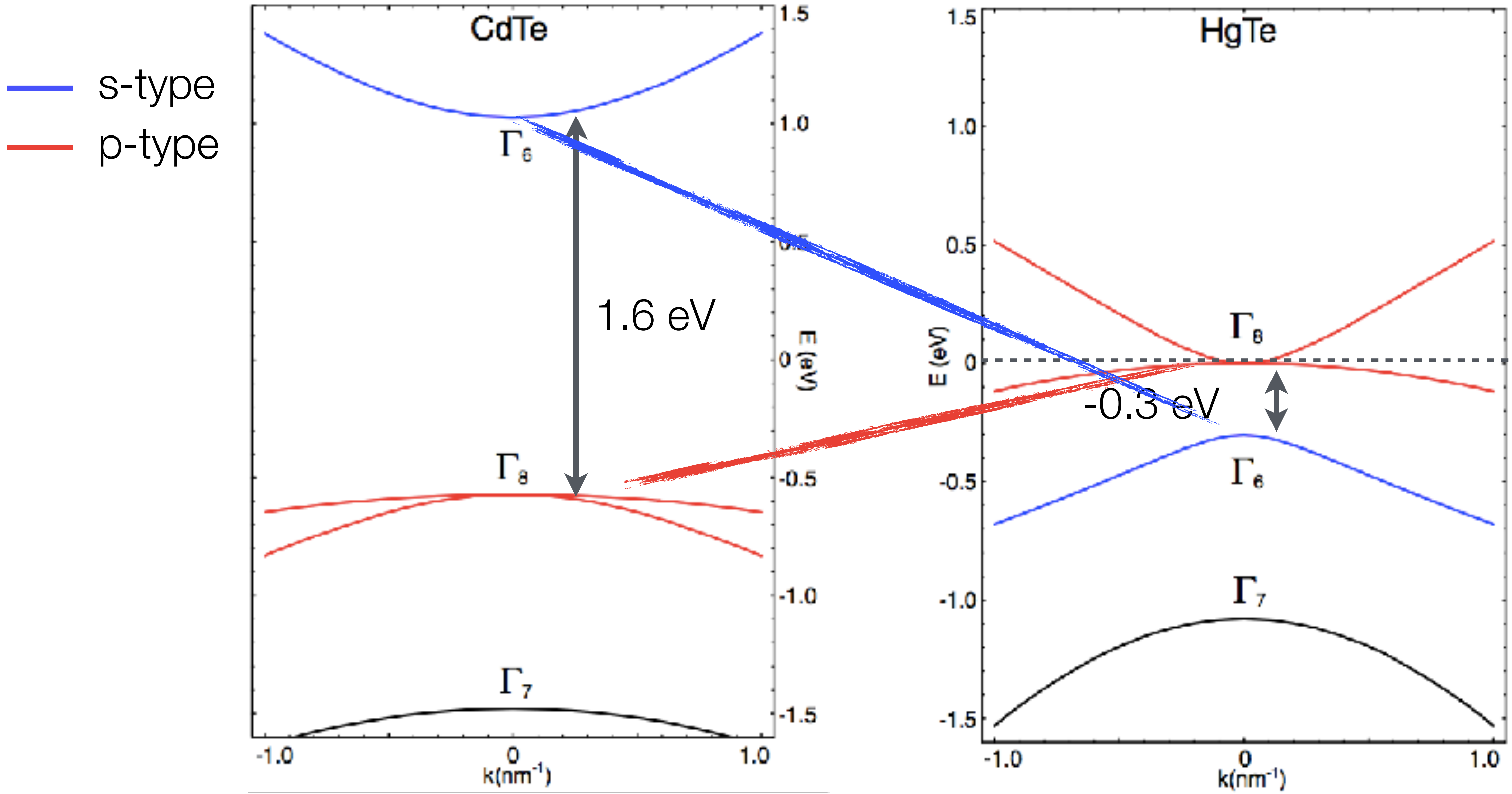
I-A. Basics of HgTe



CdTe : normal ordering
(like GaAs, Ge)

HgTe : inverted band structure

I-A. Basics of HgTe



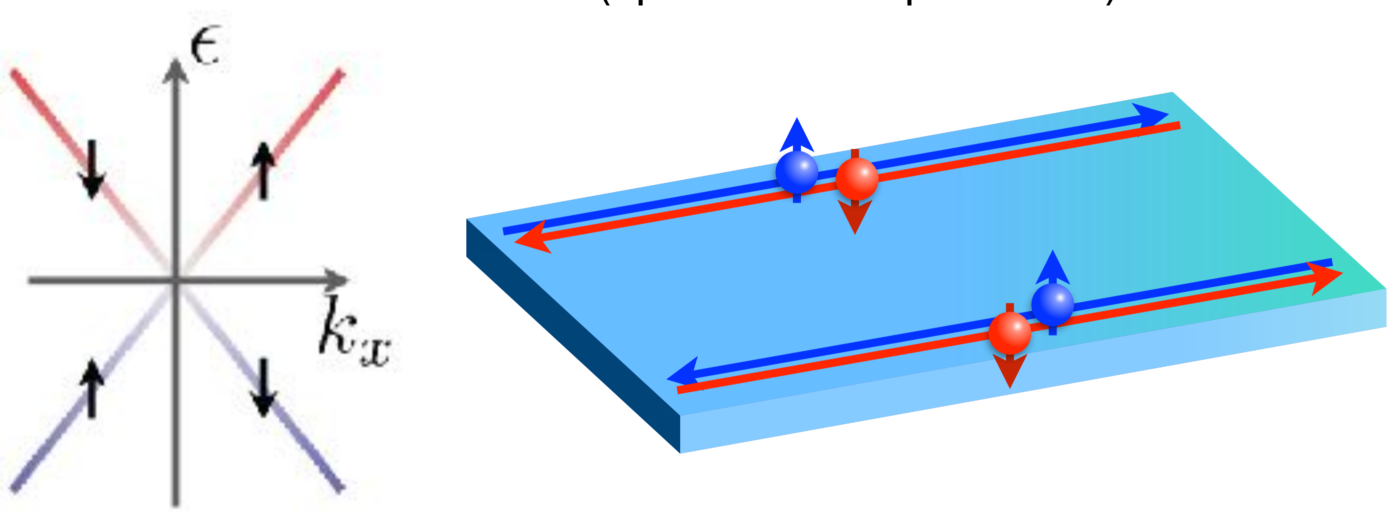
CdTe : normal ordering
(like GaAs, Ge)

HgTe : inverted band structure

- Band crossing at the interface : topological states !
- HgTe is a semi-metal

I-A. Basics of HgTe

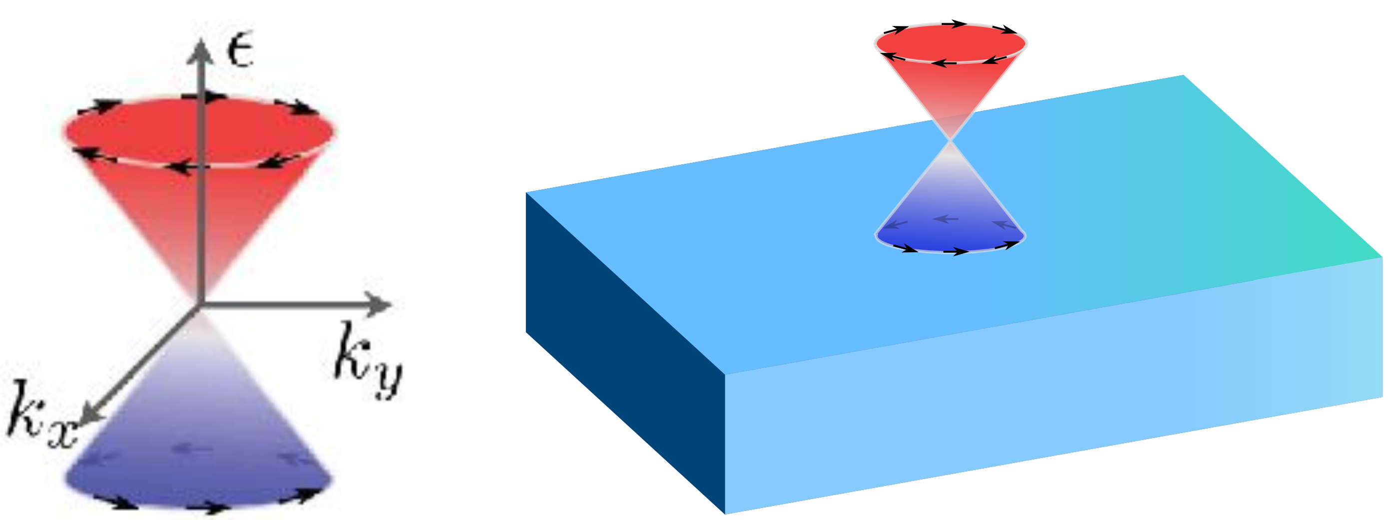
2D TI (quantum spin Hall)



Gap from quantum confinement

- narrow 2D quantum wells (<15 nm)
- 2D topological insulator
- QSH and QAH systems

3D TI (surface states)



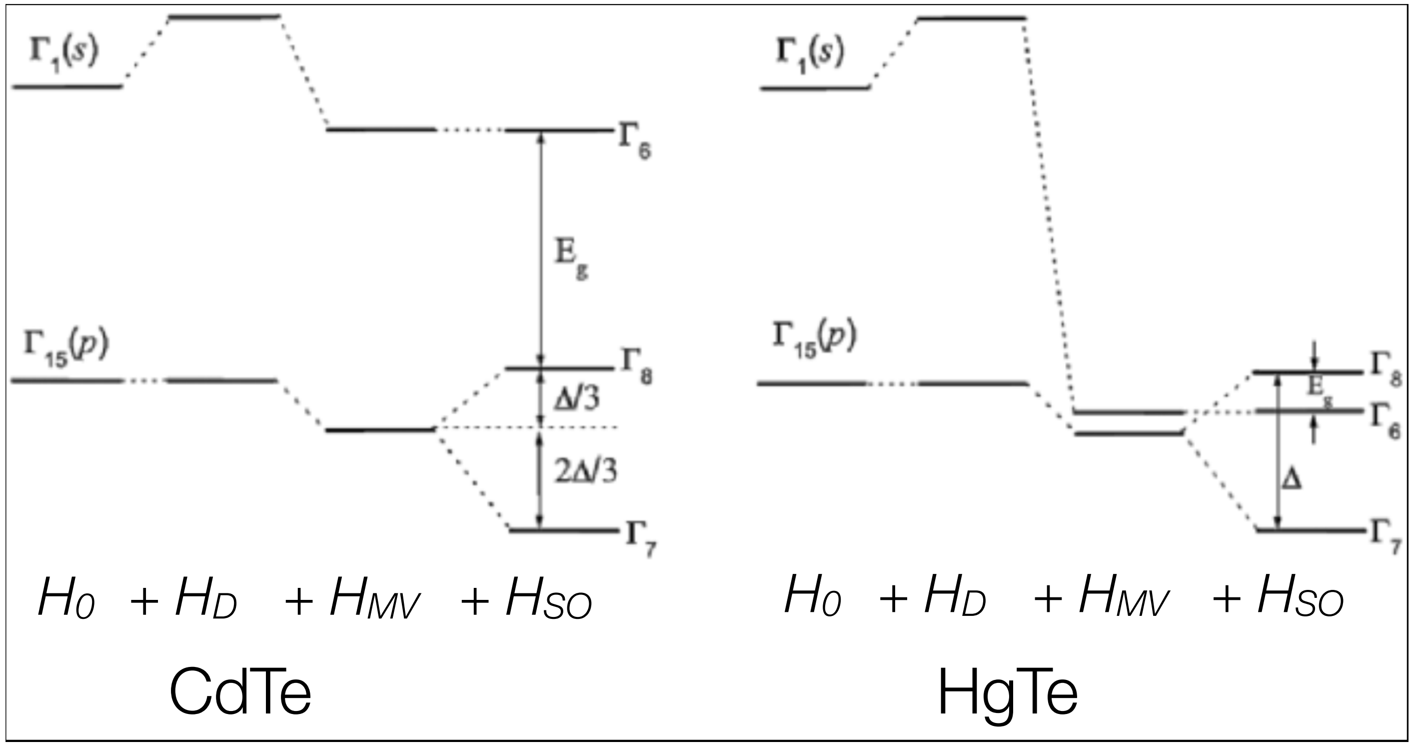
Gap from strain

- thick 3D layers (50 -150 nm)
- 3D topological insulator
- Weyl/Dirac semi-metals

I-A. Basics of HgTe

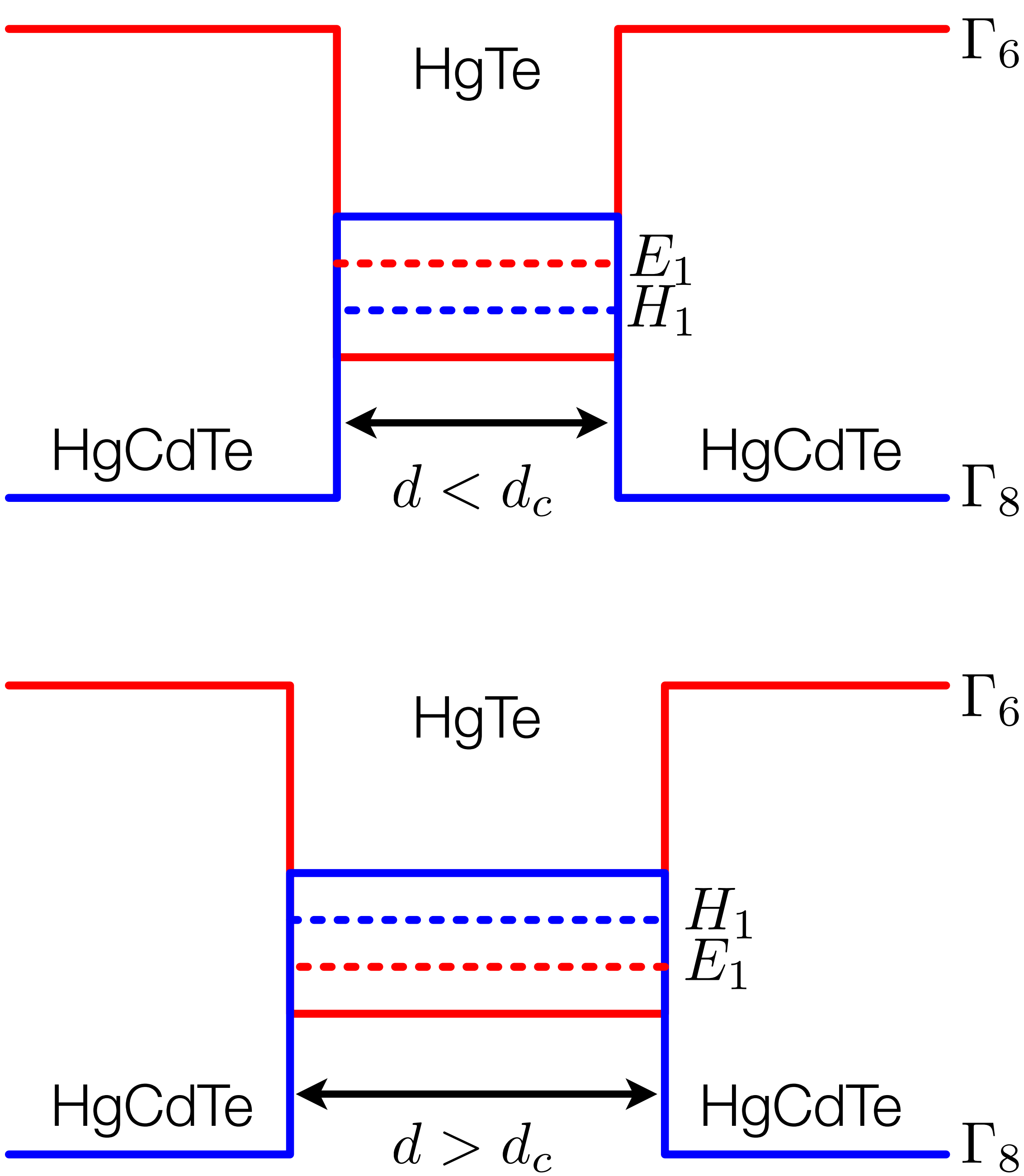
A few side remarks :

- $Hg_{0.3}Cd_{0.7}Te$ often used instead of pure CdTe (gap 0.9 eV)
- important role of mass-velocity correction ! Hg much heavier than Cd !



- surface states known since the 70s (works of Volkov-Pankratov)

I-B. Quantum wells

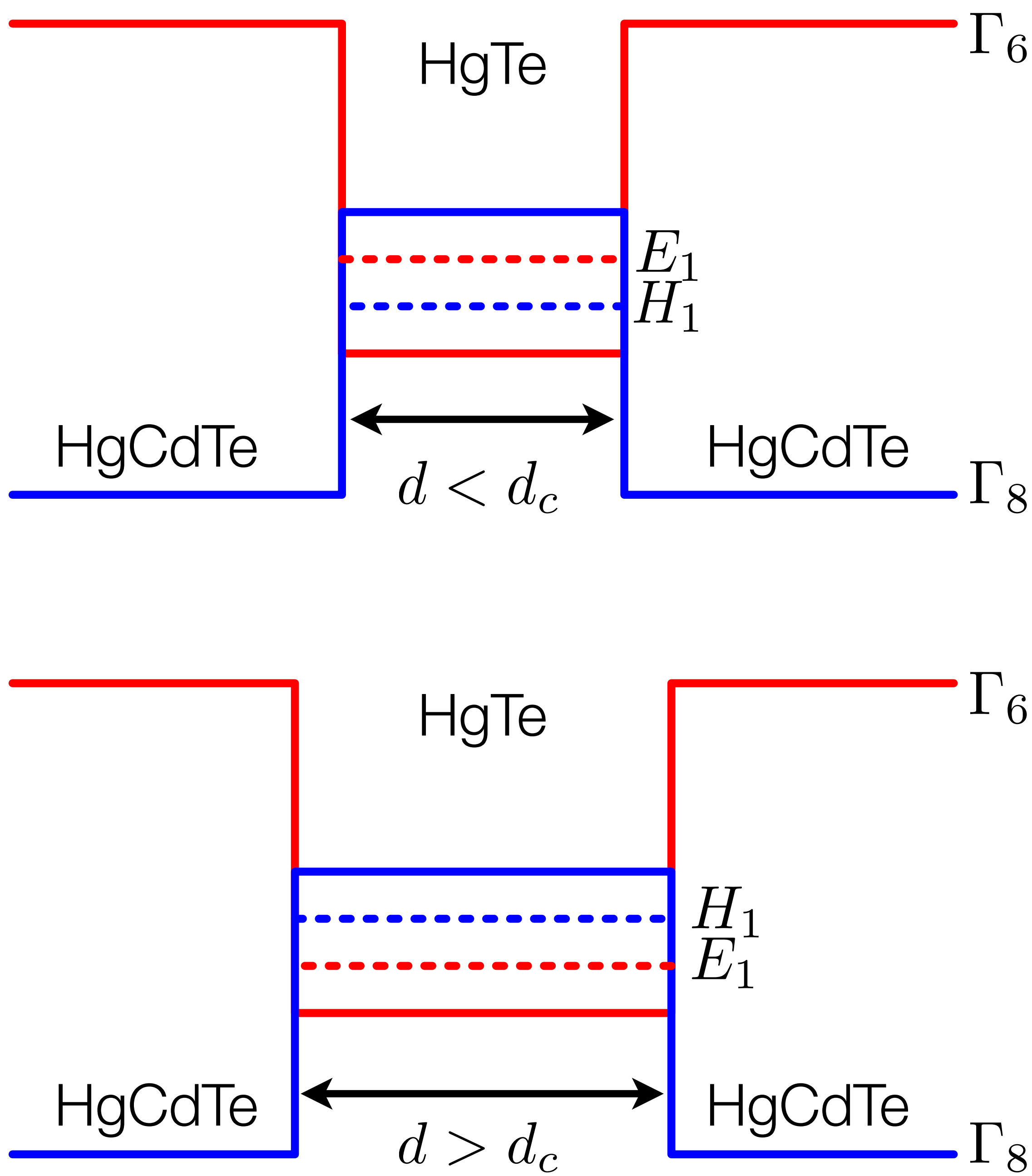


Topological phase transition

- trivial if $d < d_c$ (normal order $H_1 < E_1$)
- QSH if $d > d_c \simeq 6.3$ nm ($H_1 > E_1$)

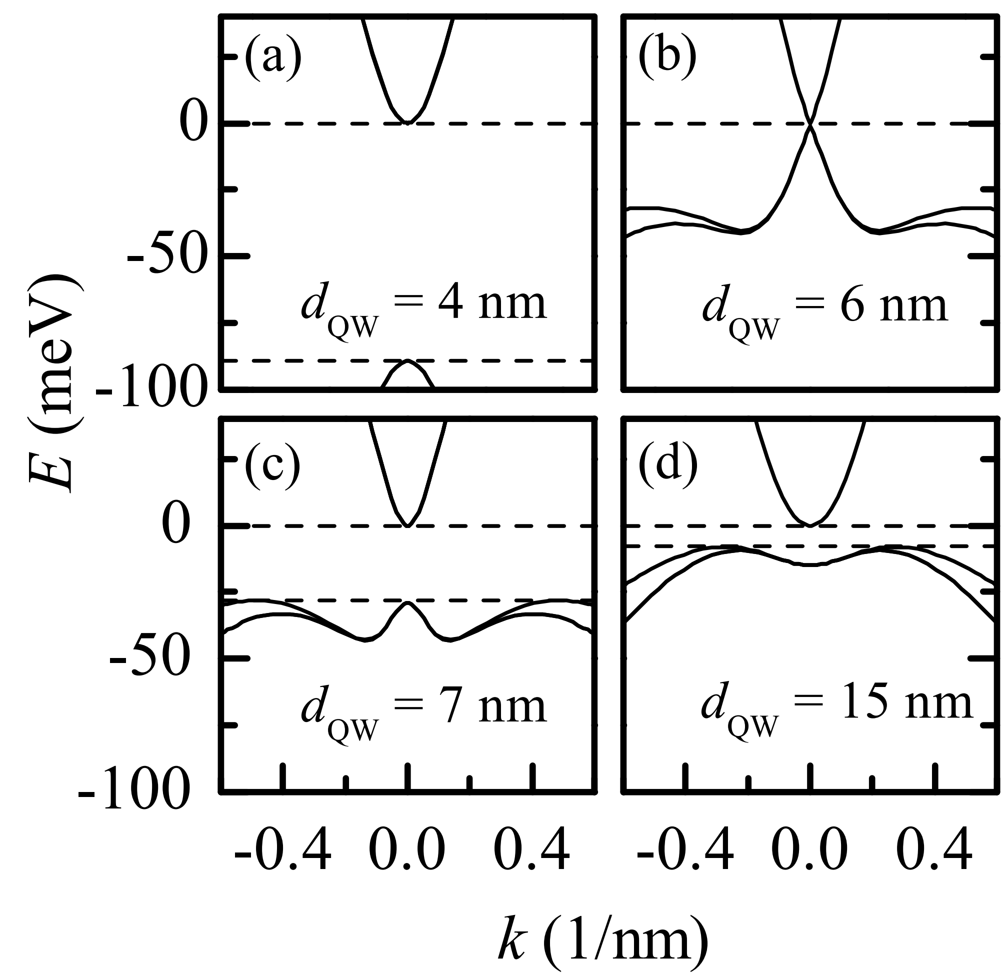
Bernevig *et al.*, Science **314**, 1757 (2006)
König *et al.*, Science **318**, 766 (2007)

I-B. Quantum wells



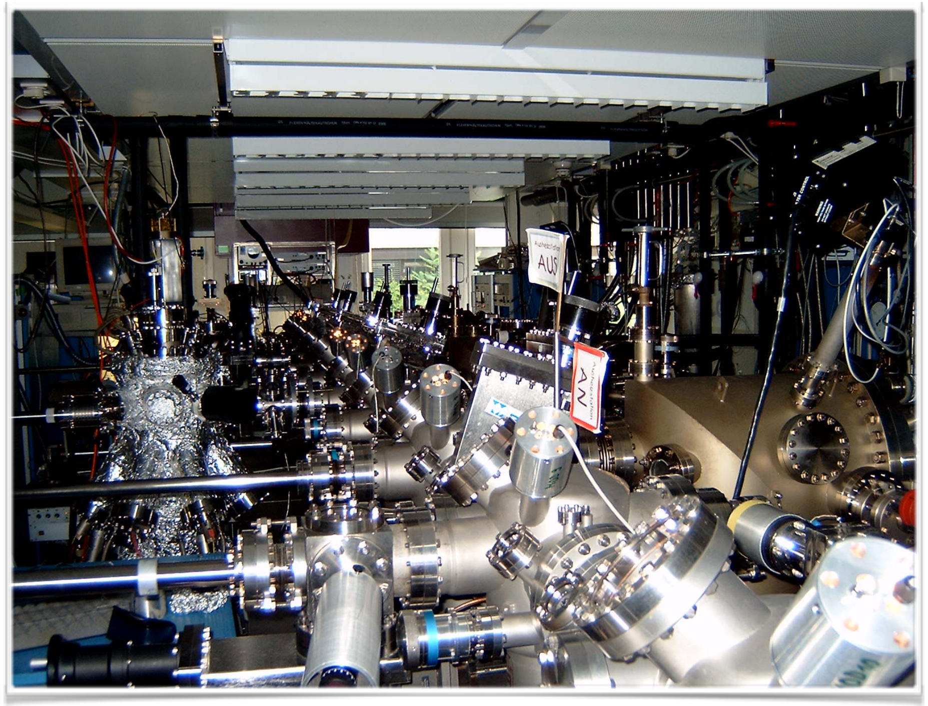
Topological phase transition

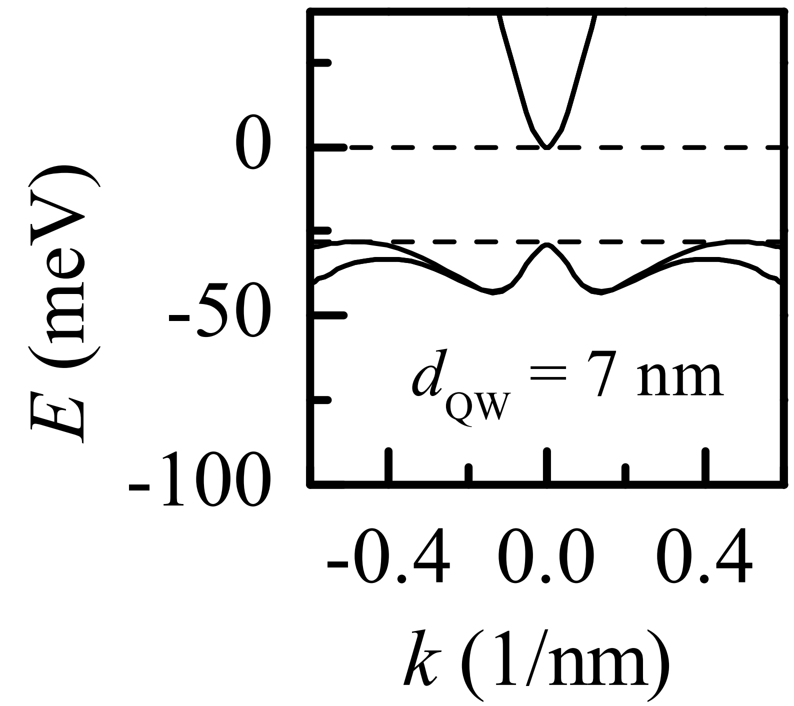
- trivial if $d < d_c$ (normal order $H_1 < E_1$)
- QSH if $d > d_c \simeq 6.3$ nm ($H_1 > E_1$)

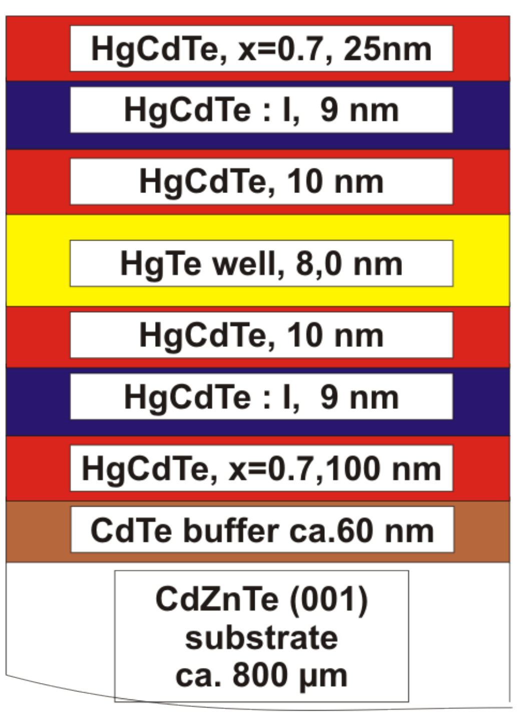


Bernevig *et al.*, Science **314**, 1757 (2006)
König *et al.*, Science **318**, 766 (2007)

I-B. Quantum wells





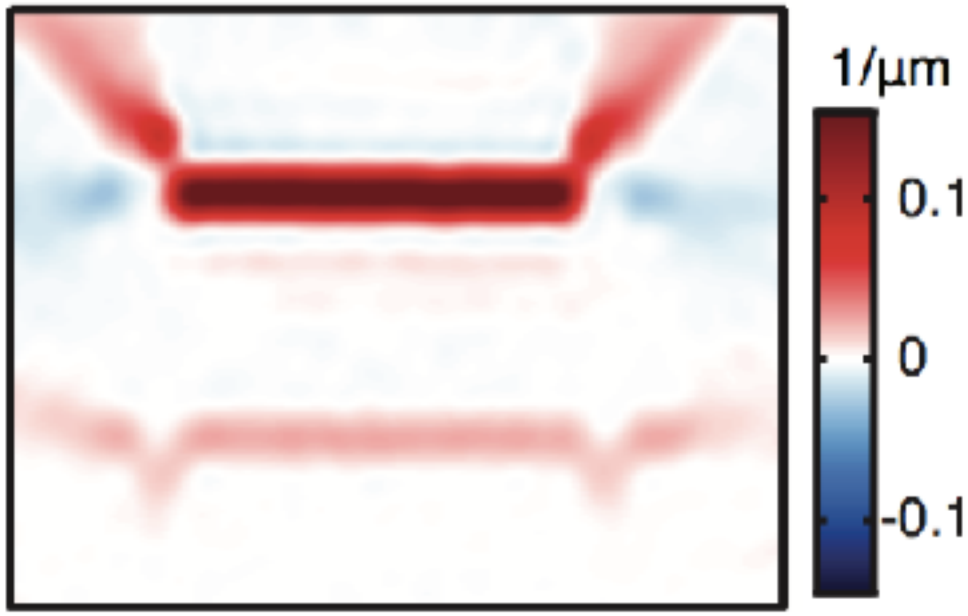


Figures of merit

- MBE growth ⇒ huge mobility
  $\mu \simeq$ 3-5 10^5 $cm^2V^{-1}s^{-1}$
- low defect density ⇒ bulk insulating
- small gap ≃ 20-25 meV
  strain engineering yields 40 meV

P. Leubner *et al.*, PRL **117**, 86403 (2016)

I-B. Quantum wells



Observations

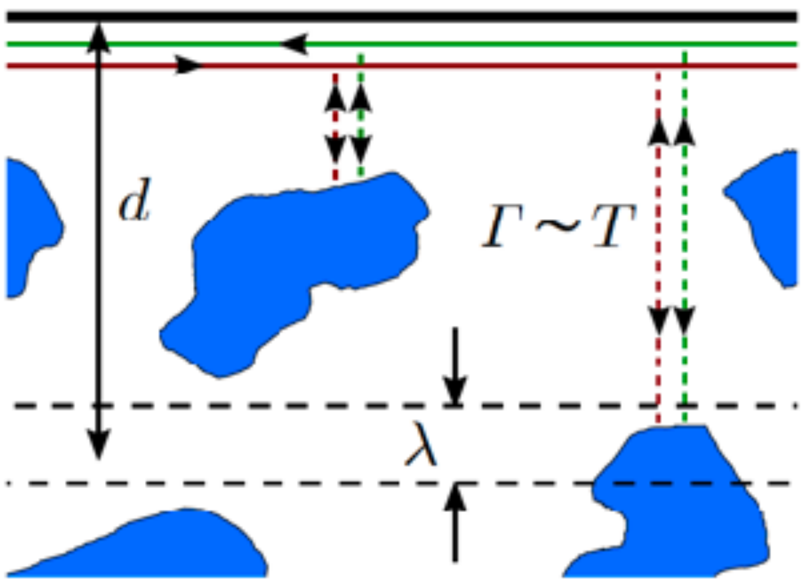
- ▹ conductance quantization (on max. 10 µm)
- ▹ non-local transport
- ▹ spin transport
- ▹ scanning SQUID imaging
- ▹ topological superconductivity?

M. König *et al.*, Science **318**, 766 (2007)
A. Roth *et al.*, Science **325**, 294 (2009)
C. Brüne *et al.*, Nat. Physics **8**, 485 (2012)
K.C. Nowack *et al.*, Nat. Materials **12**, 787 (2013)
M.K. Bendias *et al.*, submitted (2017)

Open questions - Robustness of edge states and topological properties

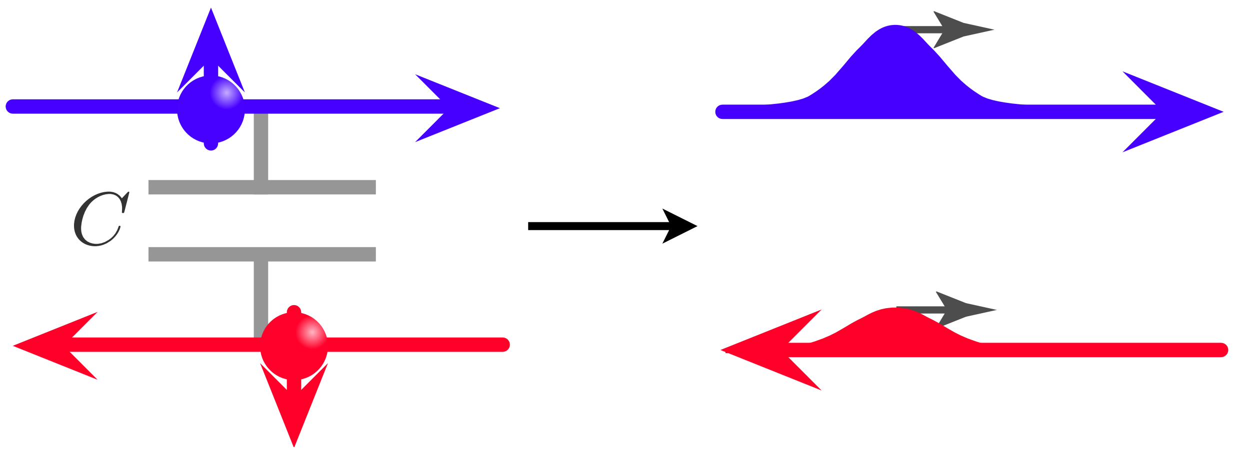
⇒ Why only 10 µm in transport ? Why not affected by bandgap ?

Disorder ? charge puddles ?



J. I. Väyrynen *et al.*, PRL **110**, 216402 (2013)

Coulomb interactions ?



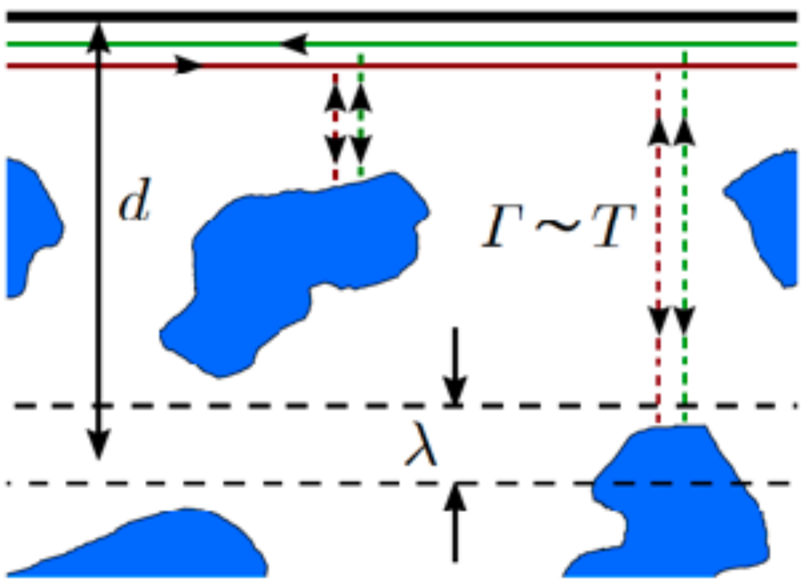
C. Wu *et al.*, PRL **96**, 106401 (2006)
G. Dolcetto *et al.*, Nuevo Cimento **39**, 113 (2015)

I-B. Quantum wells

Open questions - Robustness of edge states and topological properties

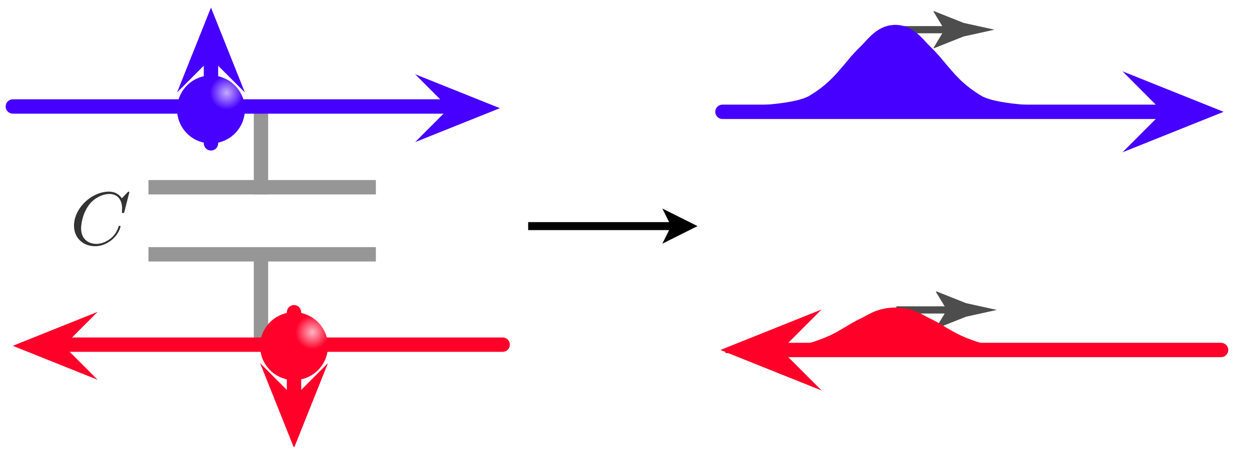
⇒ Why only 10 μm in transport ? Why not affected by bandgap ?

Disorder ? charge puddles ?



J. I. Väyrynen *et al.*, PRL **110**, 216402 (2013)

Coulomb interactions ?

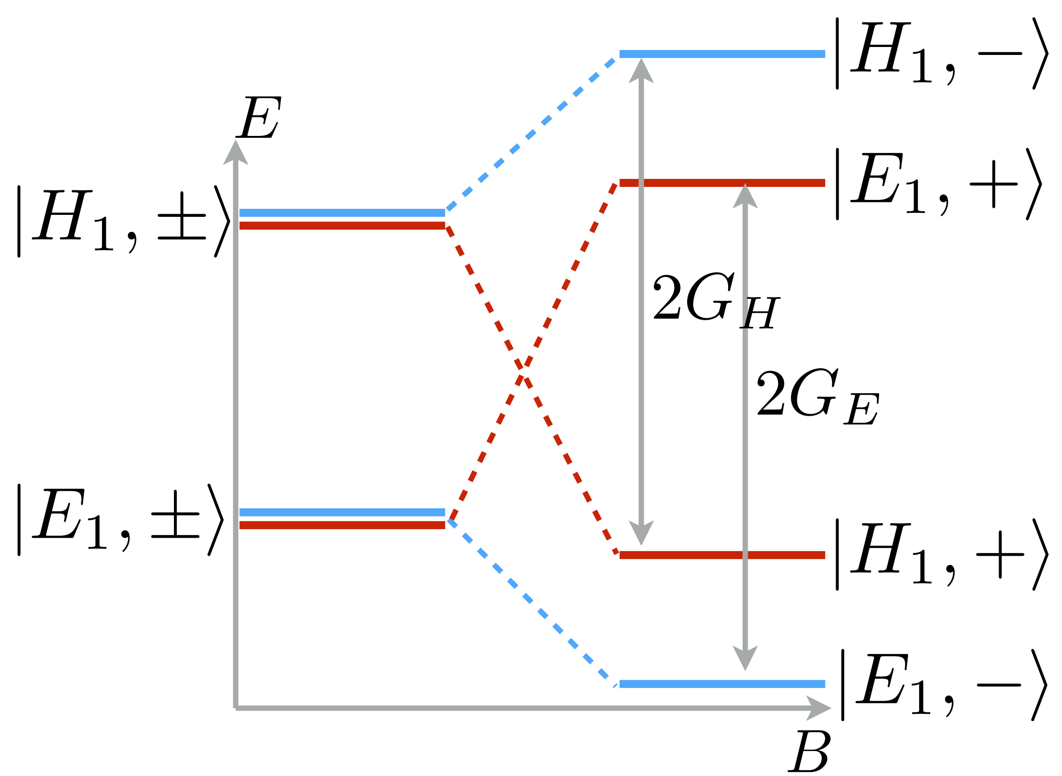


C. Wu *et al.*, PRL **96**, 106401 (2006)
G. Dolcetto *et al.*, Nuovo Cimento **39**, 113 (2015)

A long list of mechanisms that lead to backscattering…

[1] J. Maciejko *et al.*, PRL **102**, 256803 (2009)
[2] A. Ström *et al.*, PRL **104**, 256804 (2010)
[3] F. Crépin *et al.*, PRB **86**, 121106 (2012)
[4] T. L. Schmidt *et al.*, PRL **108**, 156402 (2012)
[5] F. Geissler *et al.*, PRB **89**, 235136 (2014)
[6] J. I. Väyrynen *et al.*, PRL **110**, 216402 (2013)
[7] J. I. Väyrynen *et al.*, PRB **90**, 115309 (2014)
[8] S. Essert *et al.*, PRB **92**, 205306 (2015)

I-B. QAH effect in HgTe - Theory

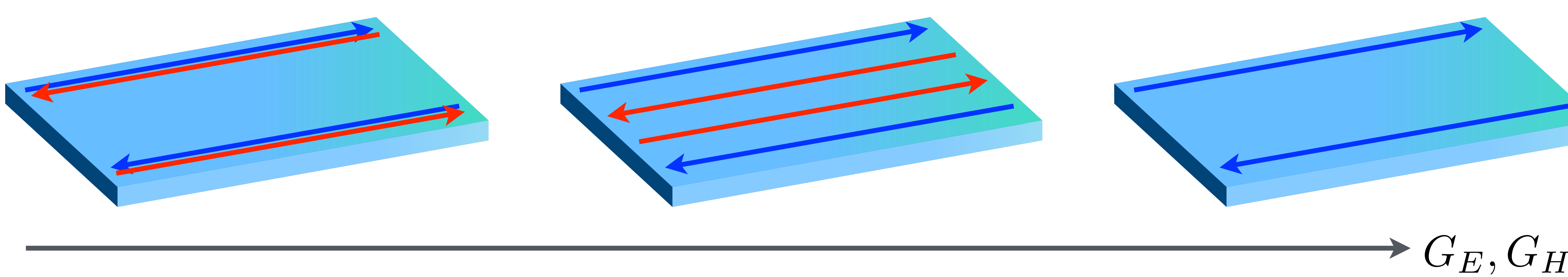


C.-X. Liu *et al.*, PRL **101**, 146802 (2008)

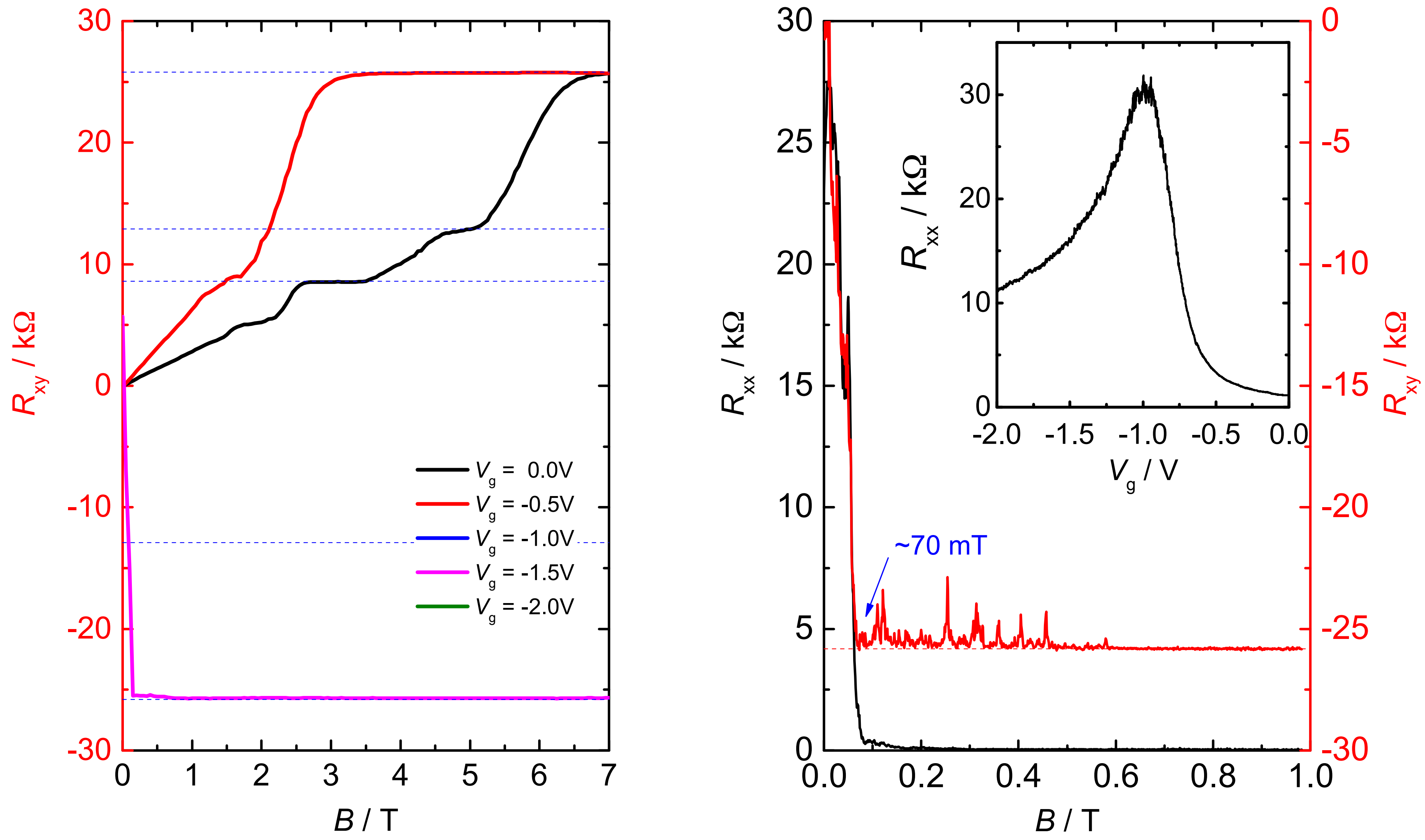
Mn-doped HgTe quantum wells

- Mn^{2+} doping (0.5 to 4 %)
  ⇒ isoelectric, paramagnetic
- 2 spin splittings with opposite signs
  $G_E, G_H \propto \langle S \rangle$
- spin polarization

$$\langle S \rangle = -\frac{5}{2} B_{5/2}\left(\frac{5}{2}\frac{g_{\mathrm{Mn}}\mu_B B}{k_B(T+T_0)}\right)$$



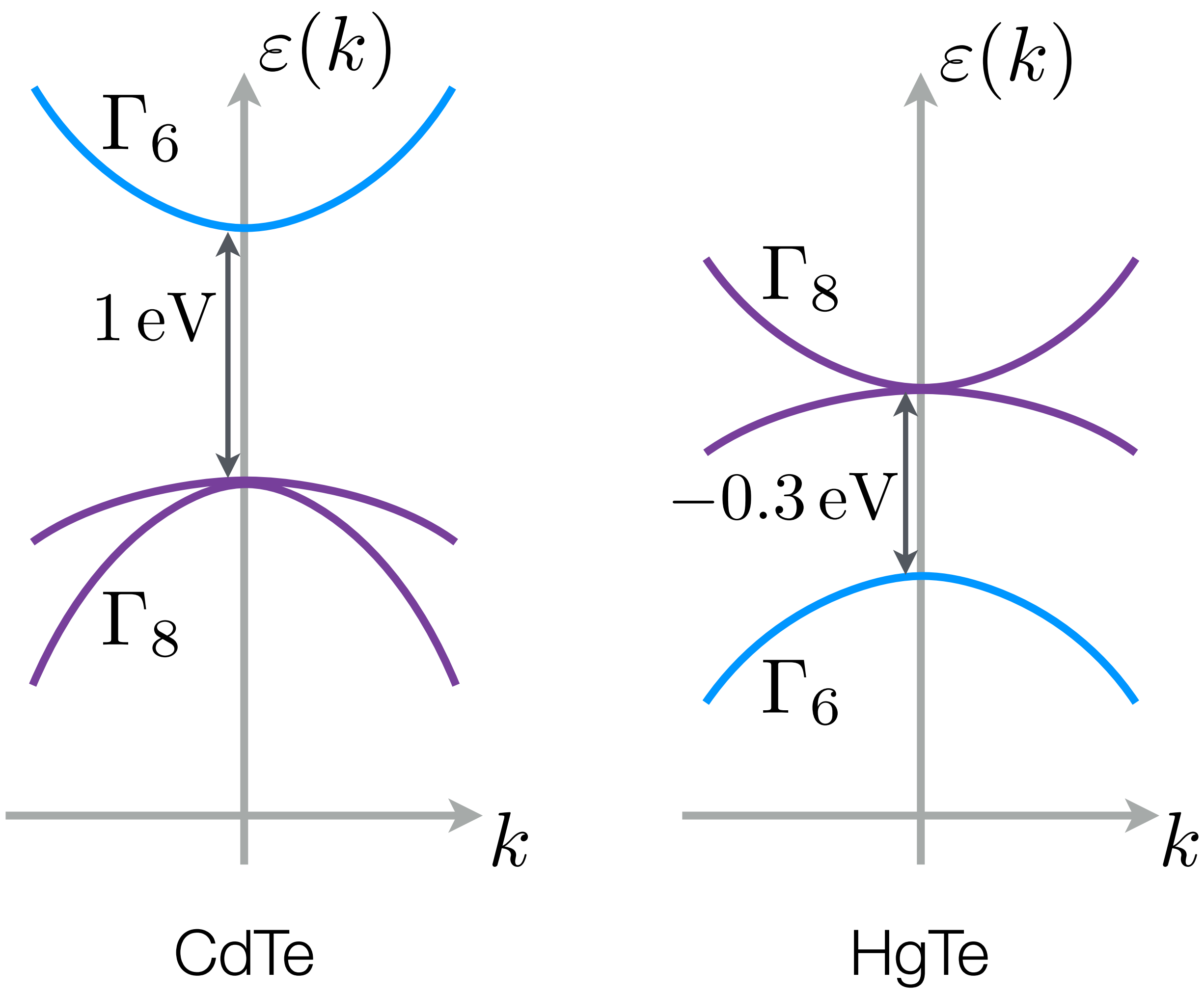
I-B. QAH effect in HgTe - Experiment



- (almost) normal QH effect in n-type regime
- early onset of ν=-1 plateau ≃70 mT (@ 30 mK)
- from T dependence, $\langle S \rangle \simeq 0.1$ at transition

Budewitz *et al.*, arXiv 1706.05789 (2017)

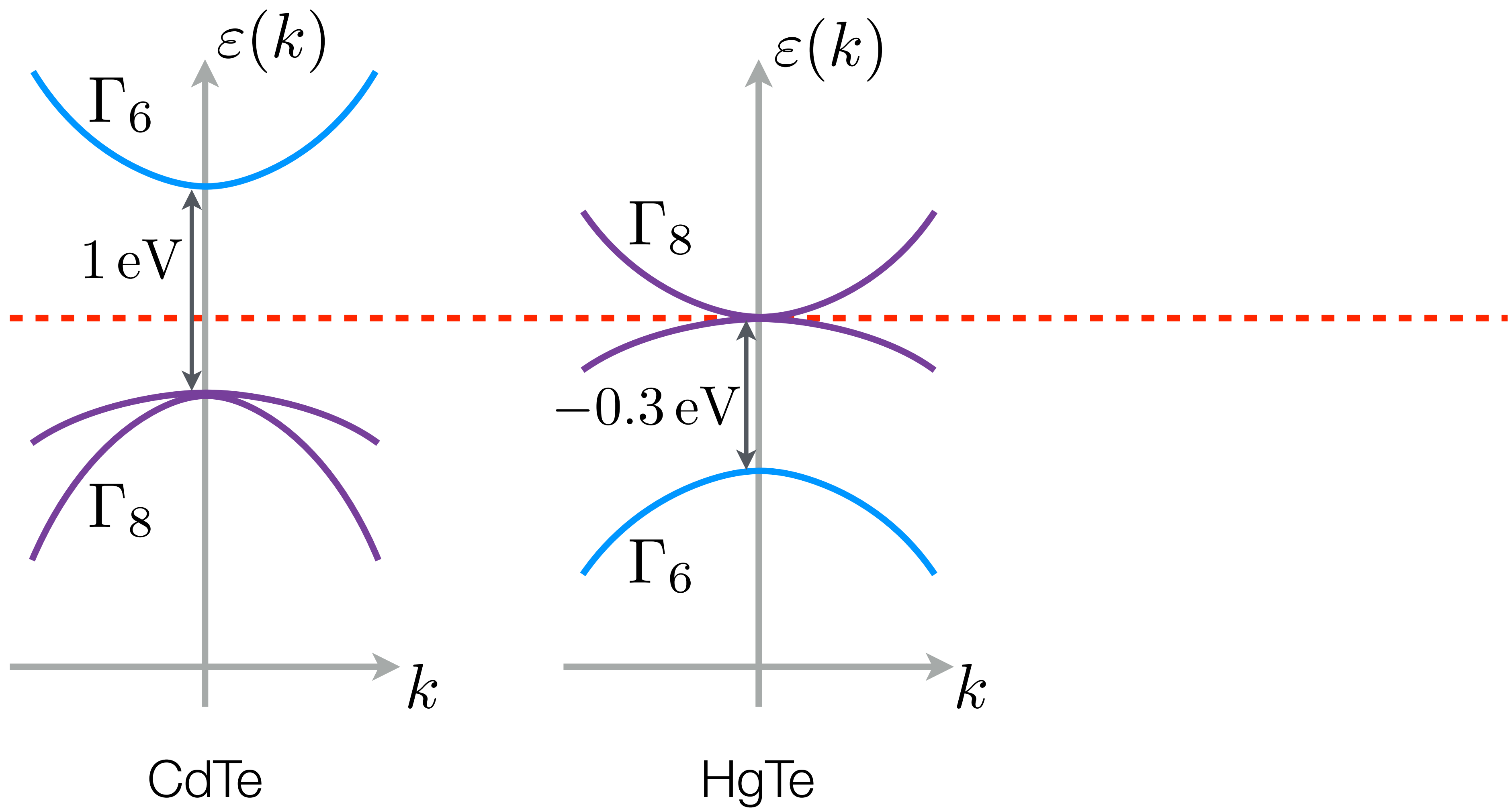
I-C. Straining HgTe 3D layers



Strained HgTe

- band inversion between Γ_6-Γ_8 ⇒ topological surface states
- strain ⇒ bulk band gap between Γ_8 subbands(20 meV)

Fu *et al.*, PRB **76**, 045302 (2007)

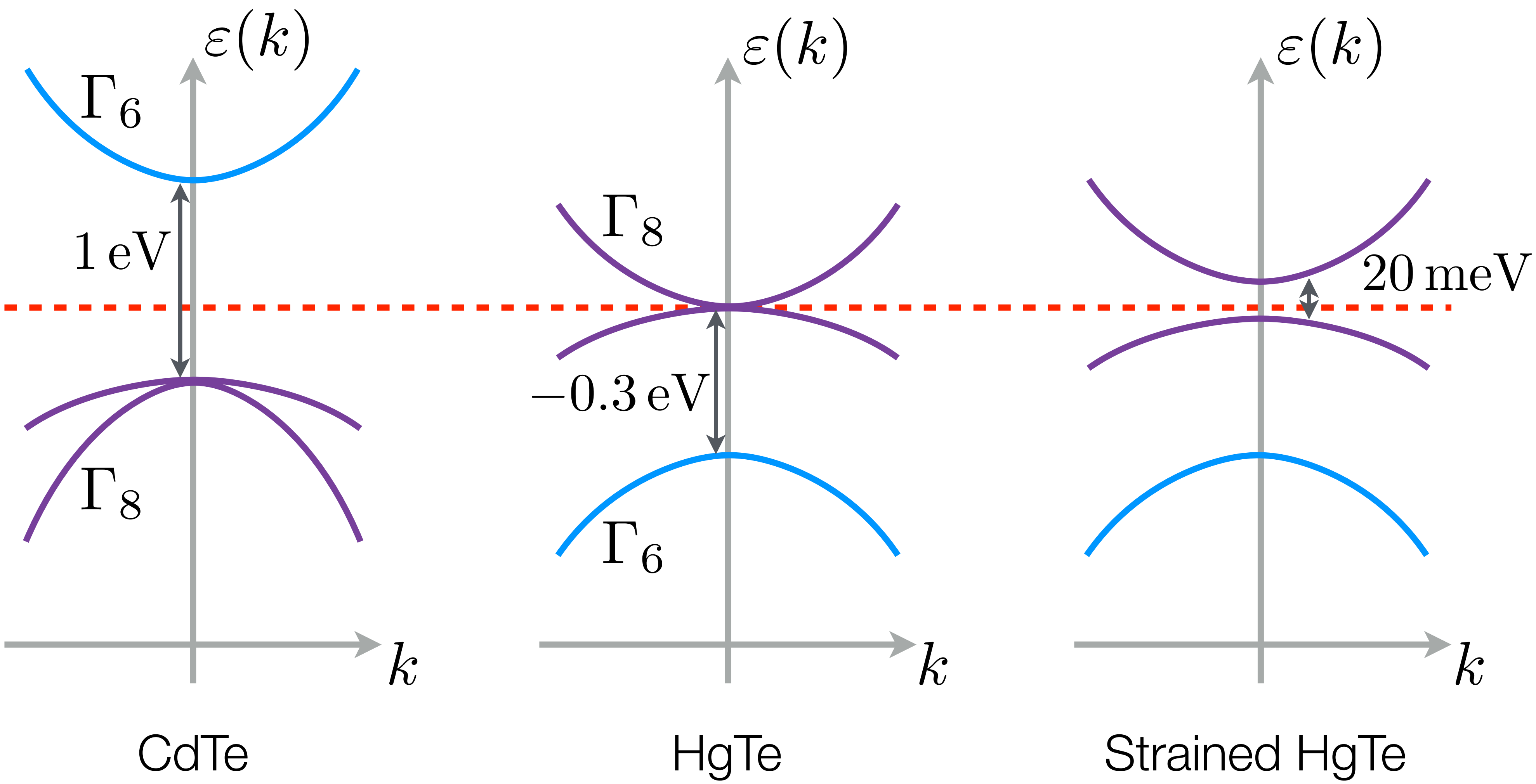


Strained HgTe

- band inversion between Γ_6-Γ_8 $\Rightarrow$ topological surface states
- strain $\Rightarrow$ bulk band gap between Γ_8 subbands(20 meV)

Fu *et al.*, PRB **76**, 045302 (2007)

I-C. Straining HgTe 3D layers

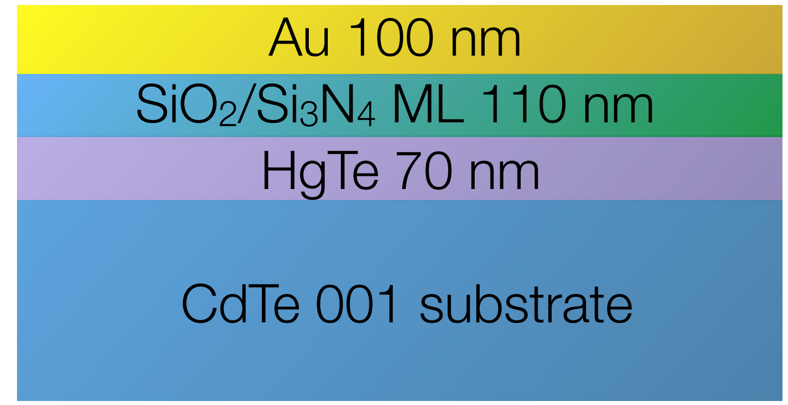


Strained HgTe

- band inversion between Γ_6-Γ_8 ⇒ topological surface states
- strain ⇒ bulk band gap between Γ_8 subbands(20 meV)

Fu *et al.*, PRB **76**, 045302 (2007)

I-C. QH effect in strained HgTe

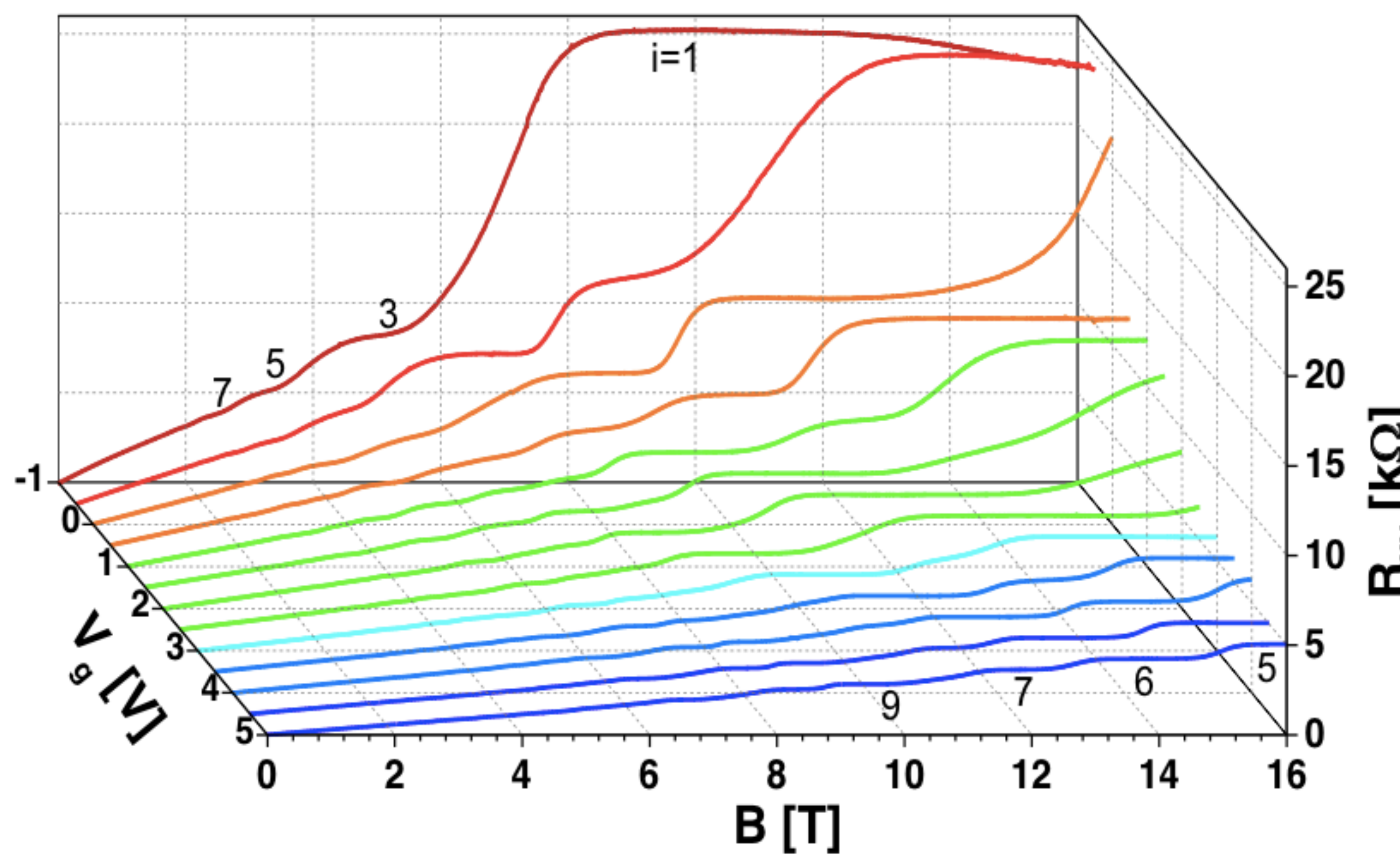


Layer structure

- ▹ HgTe (70 to 90 nm) on CdTe substrate
- ▹ Au gate on SiO_2/Si_3N_4 multilayer

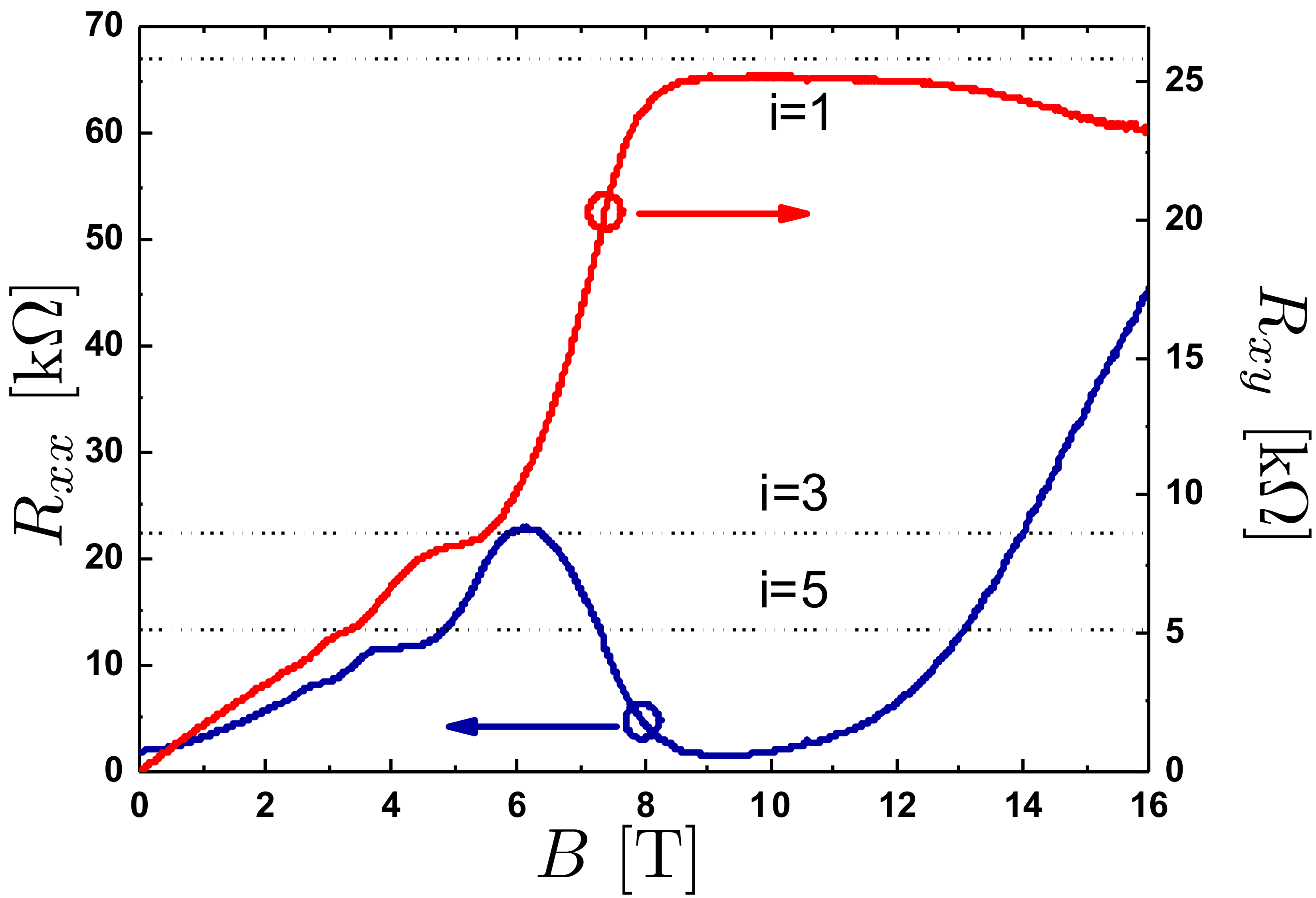
Quantum Hall effect

- ▹ electron density $n \sim 1 - 10 \times 10^{11}\,\mathrm{cm}^{-2}$
- ▹ QHE visible



Brüne *et al.*, PRL **106**, 126803 (2011)
Brüne *et al.*, PRX **4**, 041045 (2014)

I-C. Odd integer sequence



▹ Degenerate Dirac cones

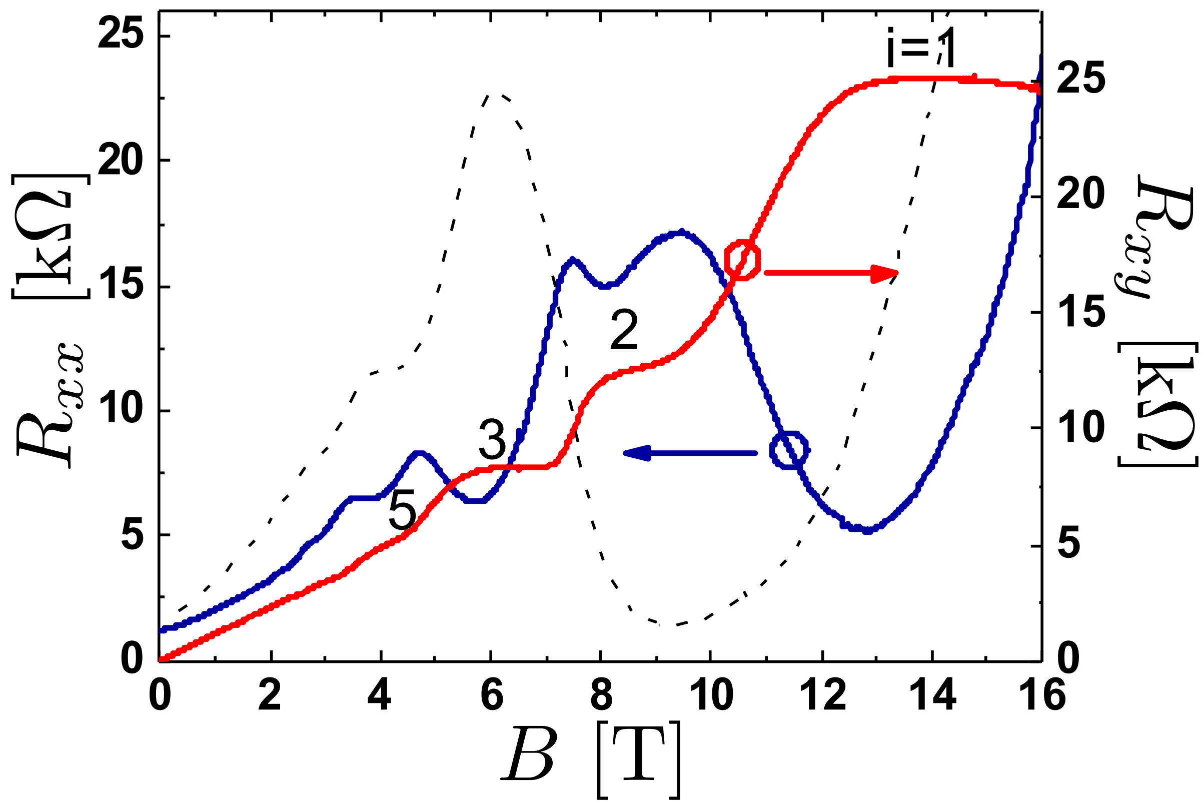
$$\nu = 2\left(n + \frac{1}{2}\right), \; n \in \mathbb{Z}$$

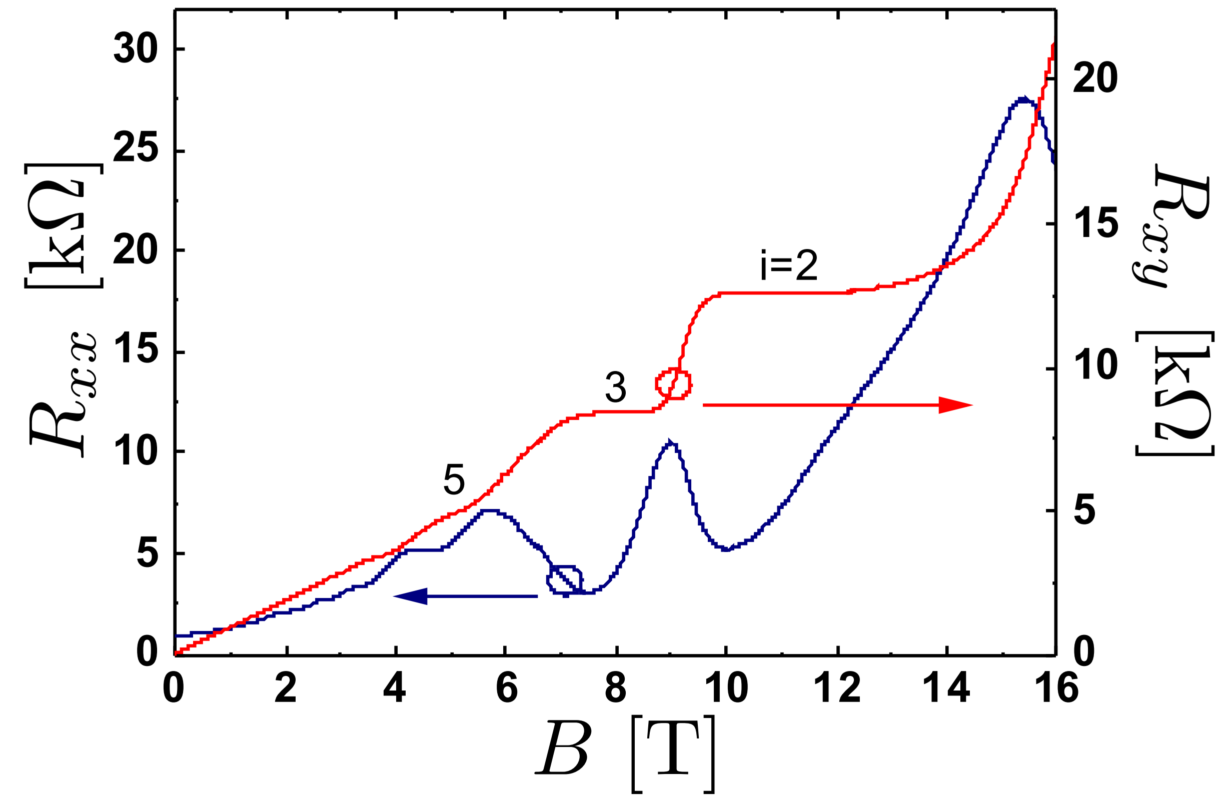
⇒ odd integers only

▹ only if densities equal

⇒ 1 value of V_g

I-C. Odd/even sequences



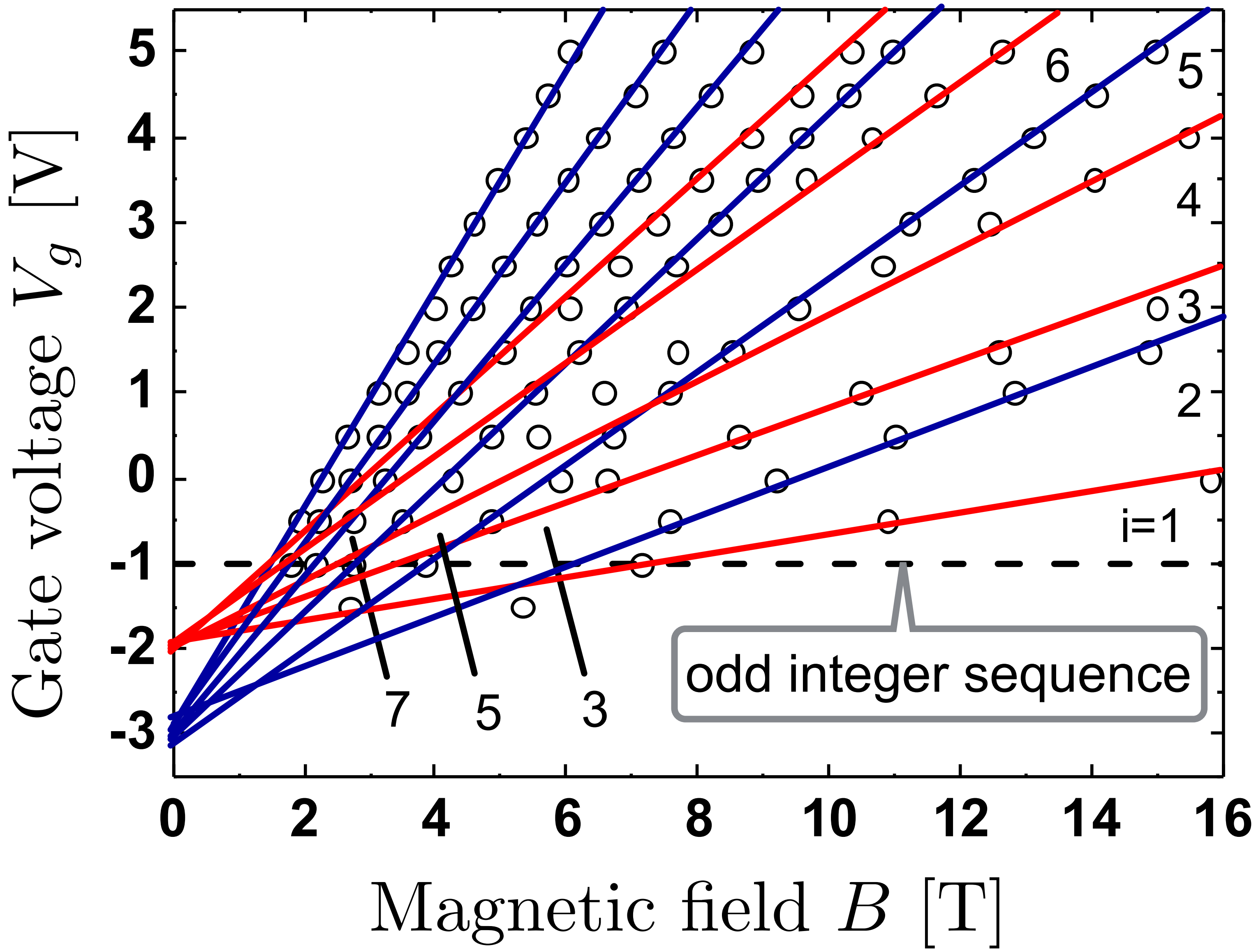


- Different densities

  $\nu = n_1 + n_2 + 1,\ n_1, n_2 \in \mathbb{Z}$ ⇒ ν=2 reappears

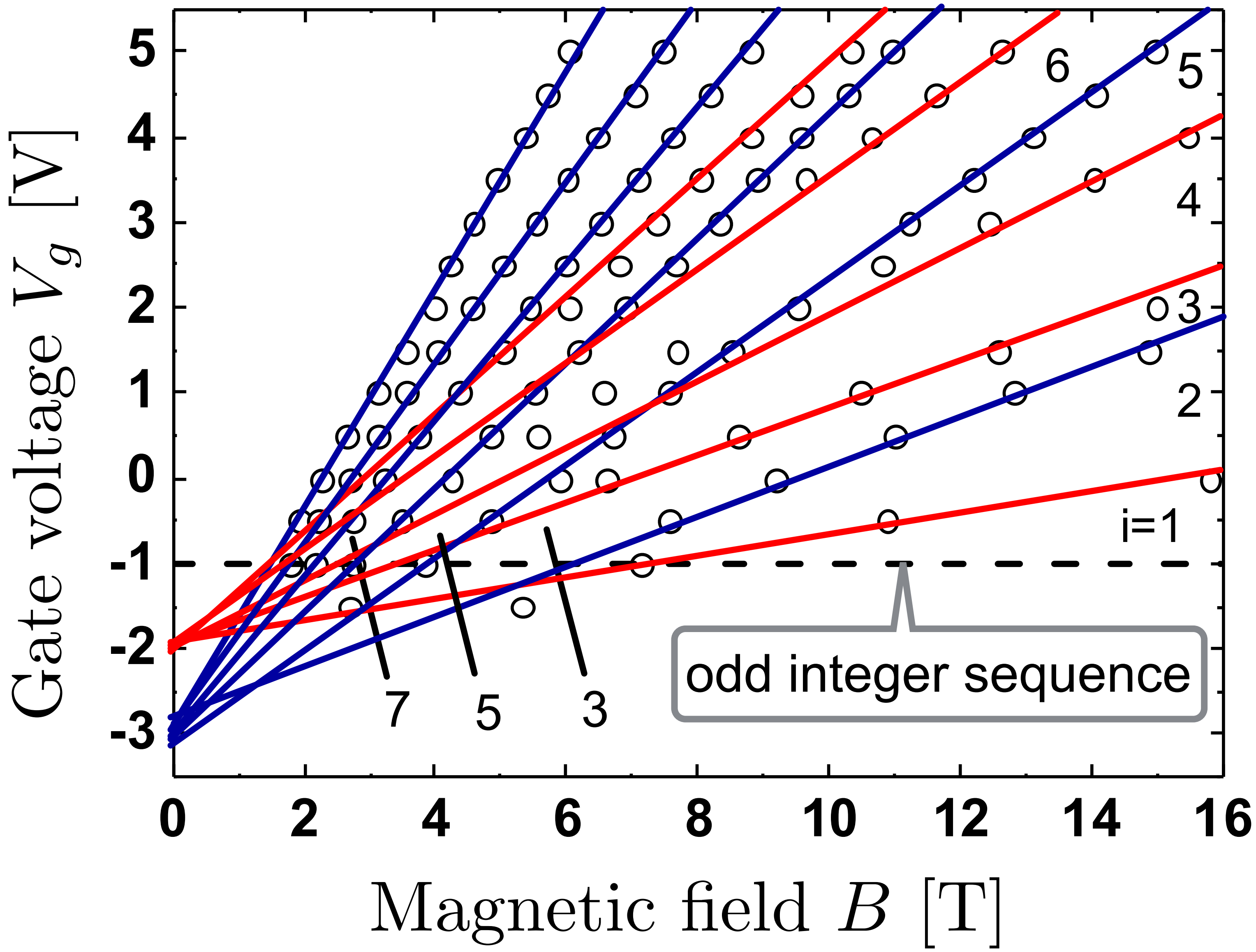
- Identification of 2 surfaces : - broadening of SdH oscillations
  - gate influence on peak positions

I-C. Tracing Landau levels



- ▹ max. of $\frac{\partial R_{xy}}{\partial B}$
- ▹ 2 subsets of LL
  - top surface
  - bottom surface

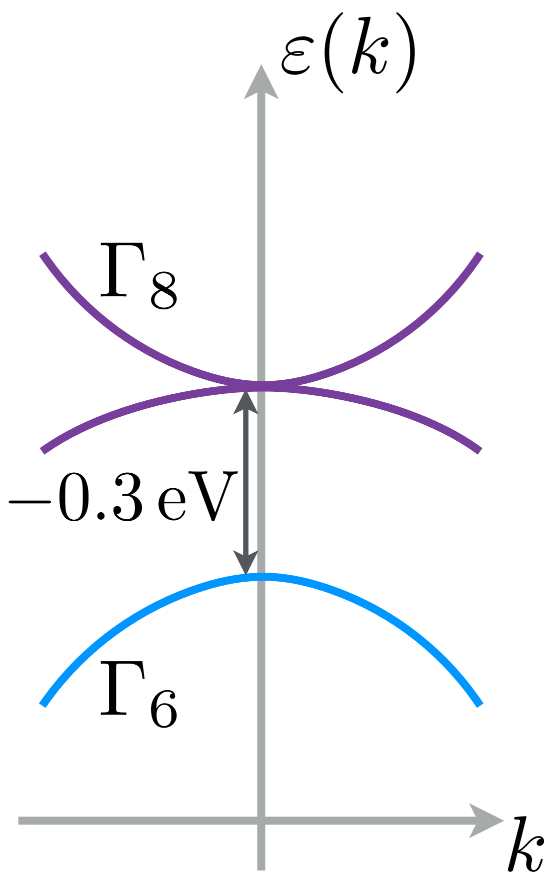
I-C. Tracing Landau levels



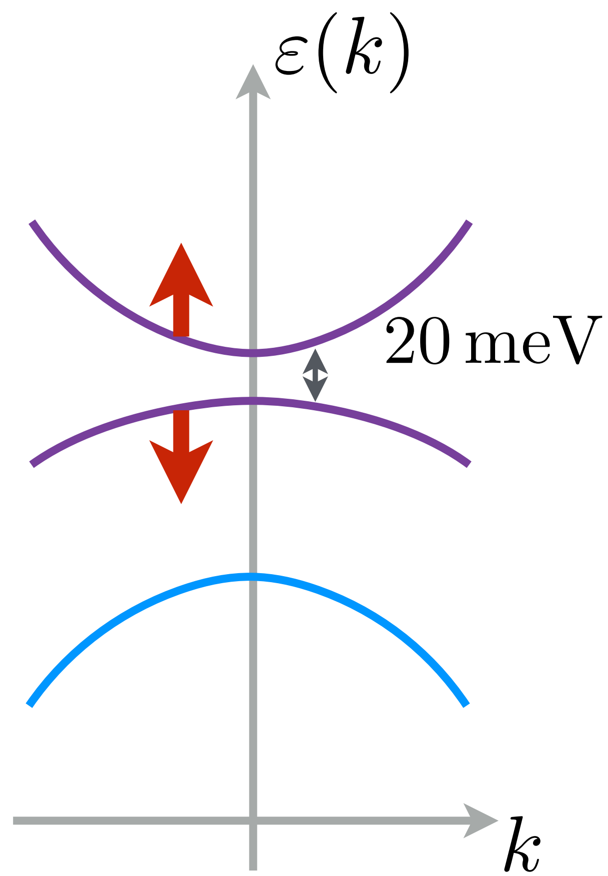
- ▹ max. of $\frac{\partial R_{xy}}{\partial B}$
- ▹ 2 subsets of LL
  - top surface
  - bottom surface

- ▹ how do surfaces/bulk exchange charge ?
- ▹ screening of the bulk by TSS ? (gating?)
- ▹ connect to one surface only ?

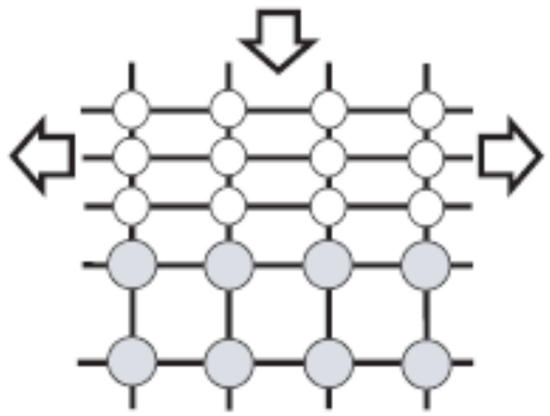
I-C. Compressive strain in 3D HgTe



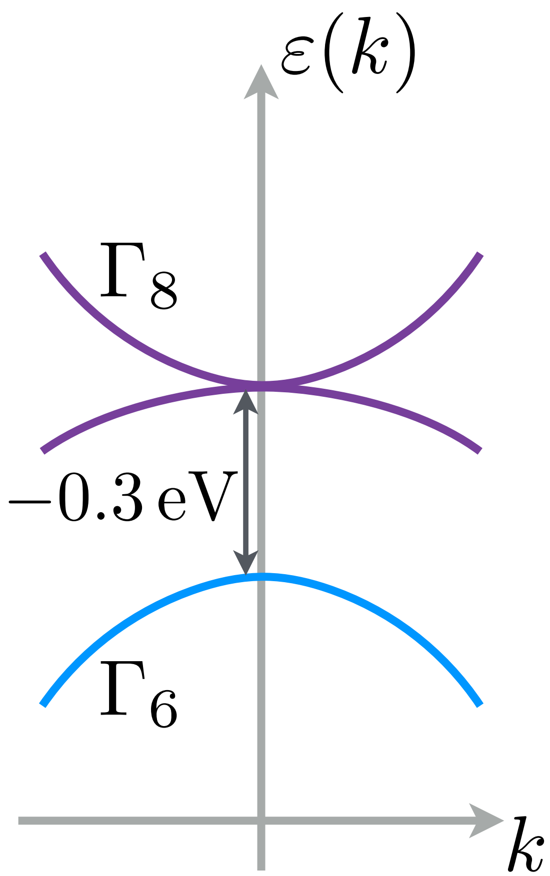
HgTe



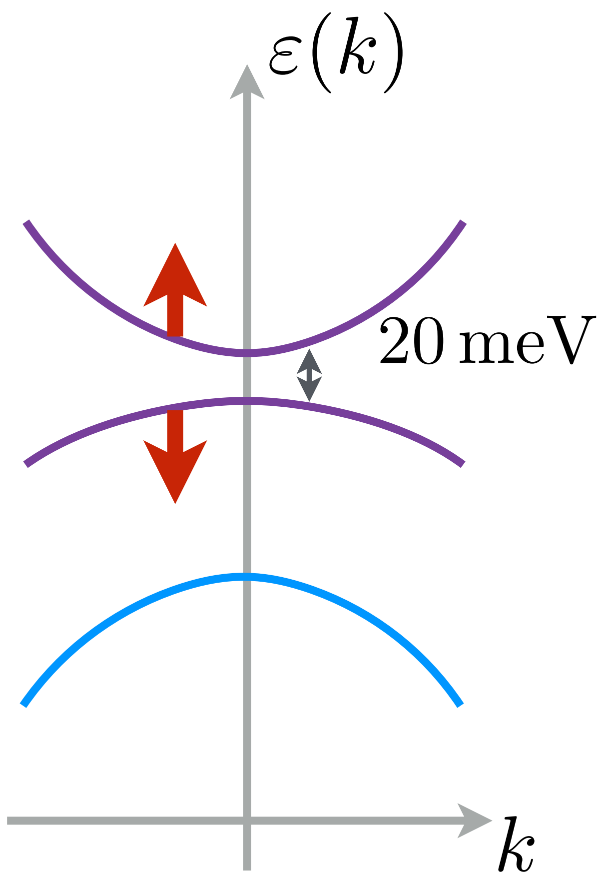
Tensile strain in HgTe



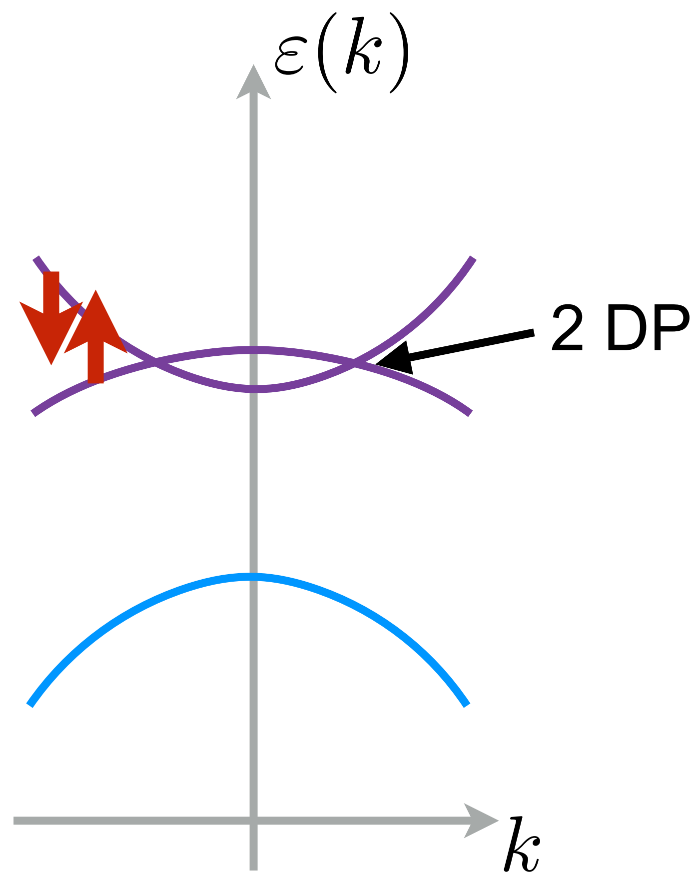
I-C. Compressive strain in 3D HgTe



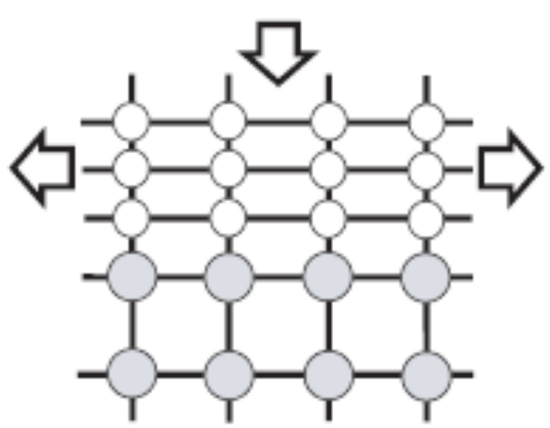
HgTe

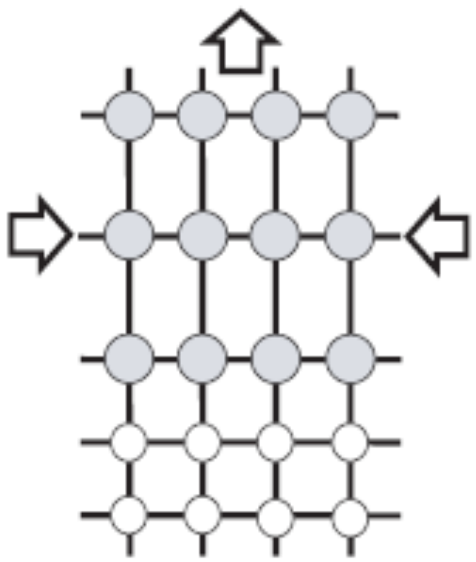


Tensile strain in HgTe

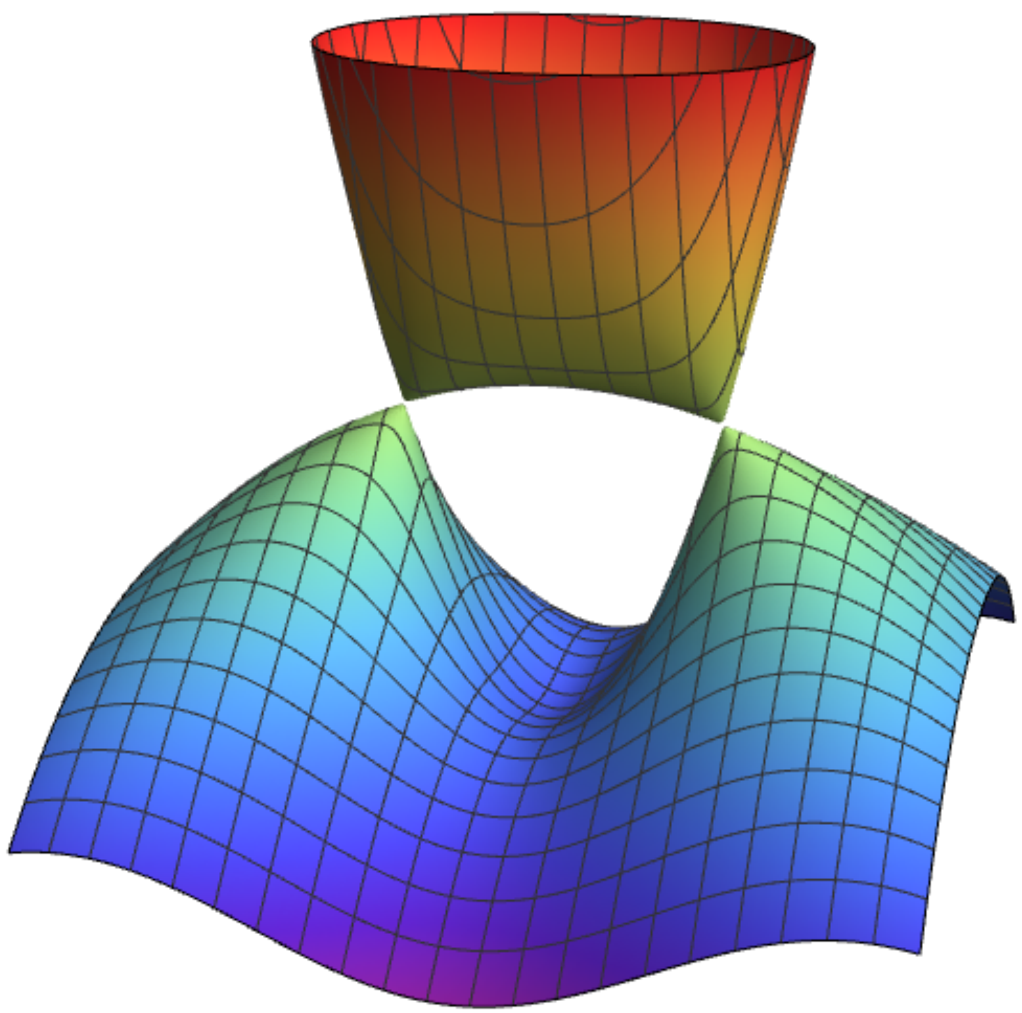


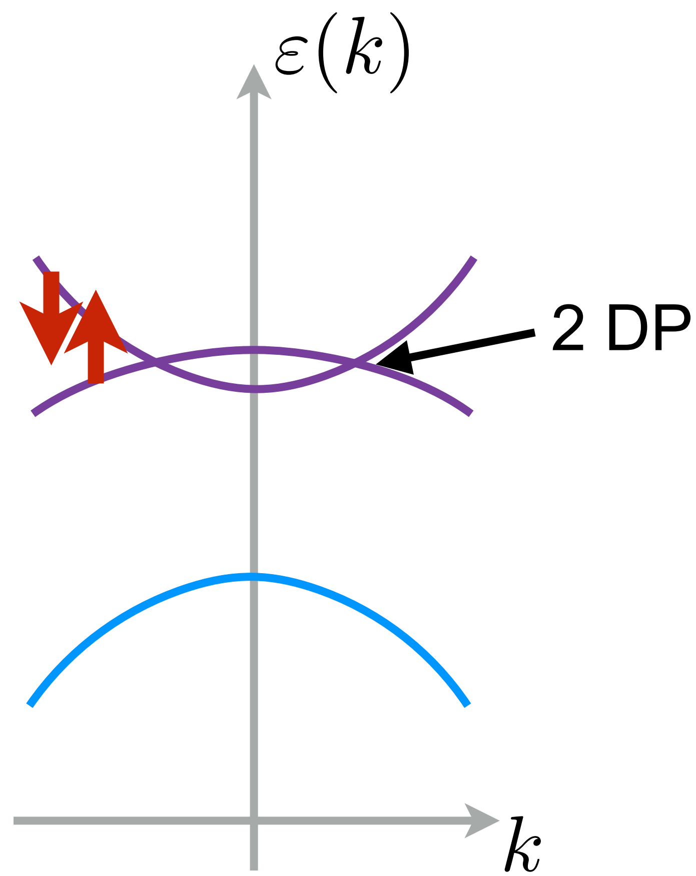
Compressive strain in HgTe



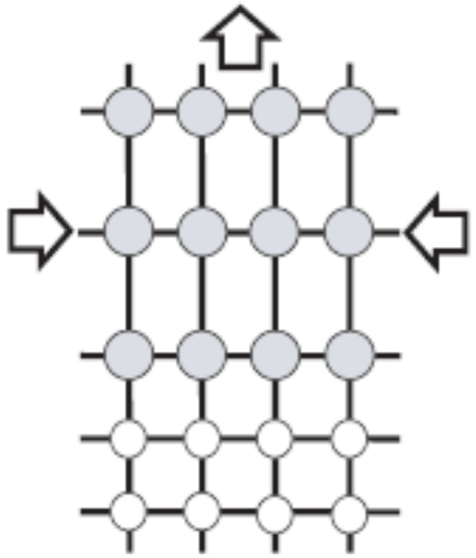


I-C. Compressive strain in 3D HgTe

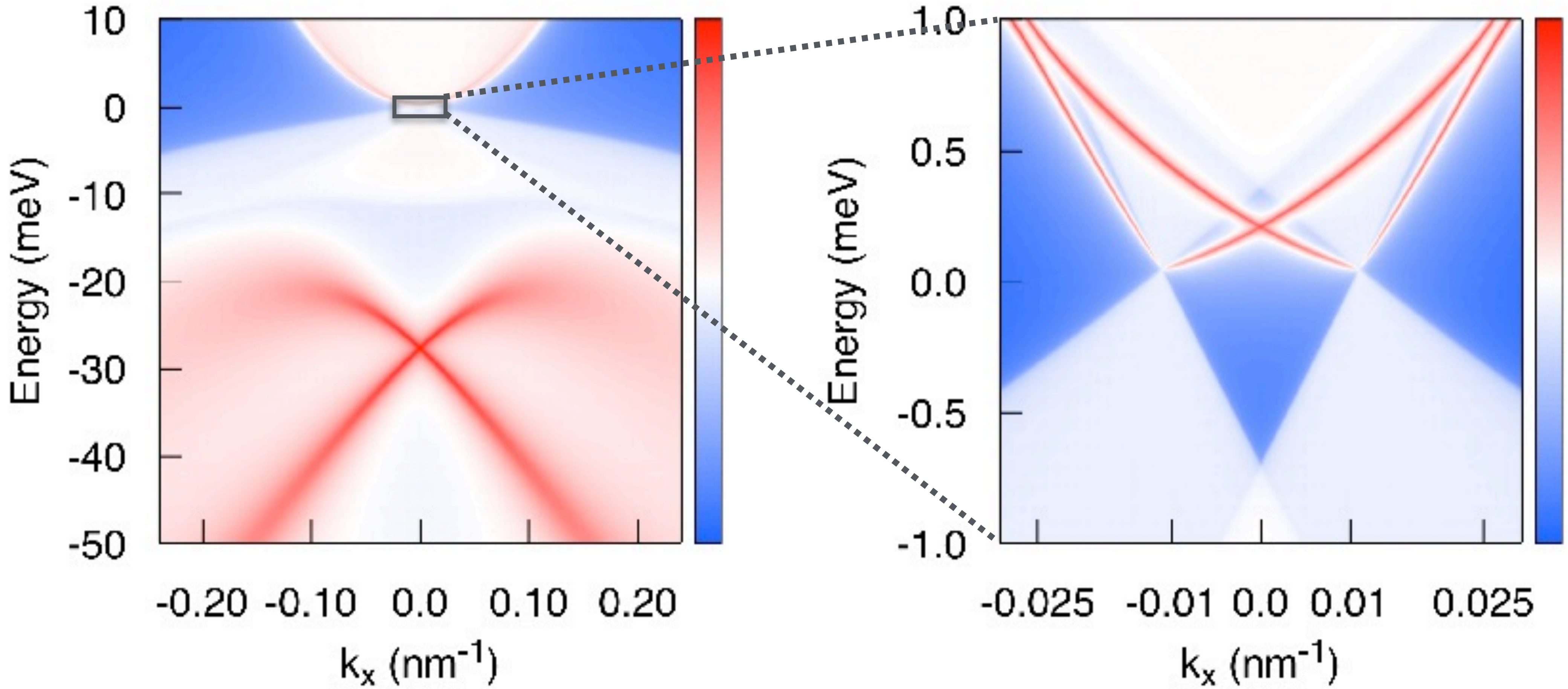




Compressive strain in HgTe



I-C. Dirac/Weyl points in HgTe



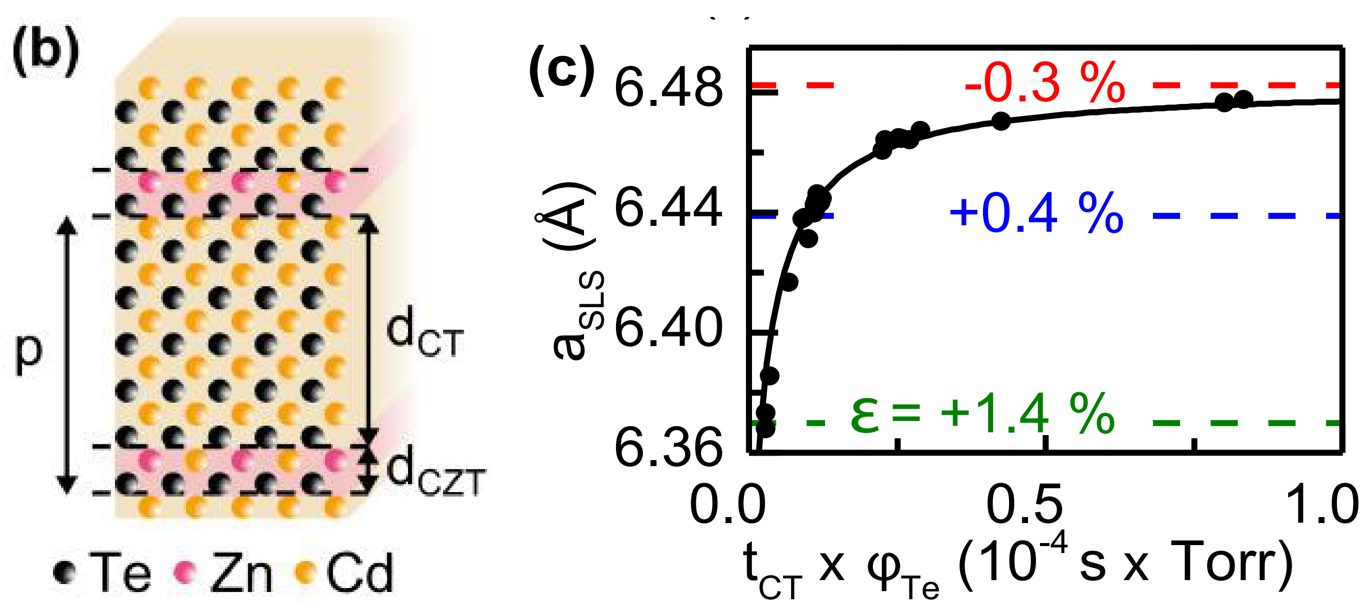
- ▹ 2 Dirac/Weyl points (degeneracy lifted by BIA)
- ▹ anticrossing of TSS states

® tiny energy scales!

J. Ruan *et al.*, Nature Comm. **7**, 11136 (2016)

DFT calculations : D. Di Sante, G. Sangiovanni, R. Thomale and E.M. Hankiewicz (Würzburg)

I-C. Strain engineering



P. Leubner *et al.*, PRL **117**, 86403 (2016)

Virtuel substrates

- superlattice with:
  - 1 layer of $Cd_{0.5}Zn_{0.5}Te$
  - x layers of CdTe
- effective lattice parameter

$$a_{\rm eff} = a_{\rm CdTe}\left(1 + \frac{f}{1+mr}\right)$$

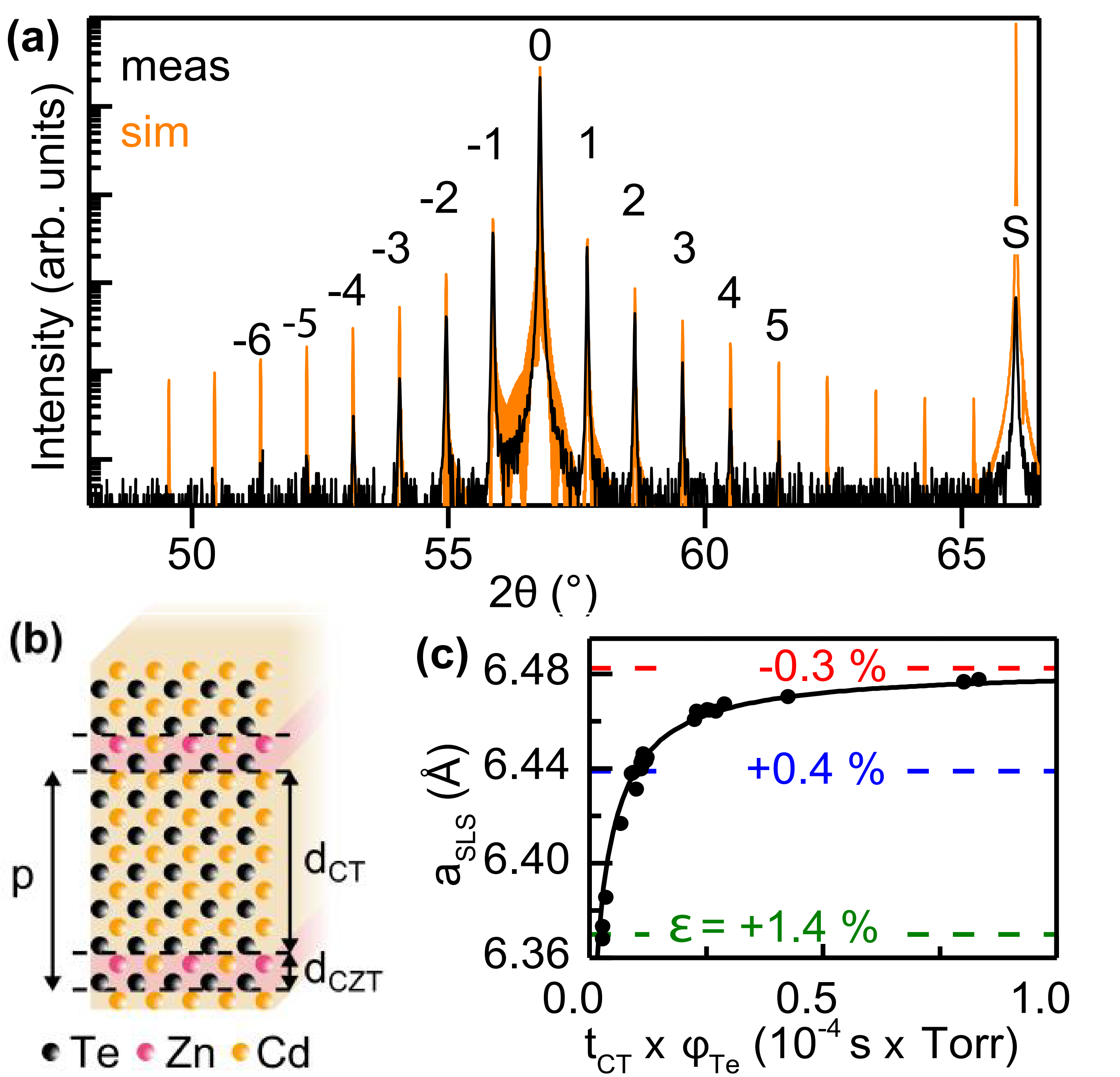
f lattice mismatch
m stiffness ratio
r thickness ratio

$$a_{\rm HgTe} = 0.646\,{\rm nm}$$

$$a_{\rm CdTe} = 0.648\,{\rm nm}$$

$$a_{\rm ZnTe} = 0.610\,{\rm nm}$$

I-C. Strain engineering



P. Leubner *et al.*, PRL **117**, 86403 (2016)

Virtuel substrates

- superlattice with:
  - 1 layer of $Cd_{0.5}Zn_{0.5}Te$
  - x layers of CdTe
- effective lattice parameter

$$a_{\mathrm{eff}} = a_{\mathrm{CdTe}}\left(1 + \frac{f}{1+mr}\right)$$

f lattice mismatch
m stiffness ratio
r thickness ratio

$$a_{\mathrm{HgTe}} = 0.646\,\mathrm{nm}$$
$$a_{\mathrm{CdTe}} = 0.648\,\mathrm{nm}$$
$$a_{\mathrm{ZnTe}} = 0.610\,\mathrm{nm}$$

Summary of Part I

Versatility of HgTe platform

- several topological systems possible:
  2D/3D TIs, Weyl semi-metal, QAH/QH/QSH + core-shell NW

- high quality samples:
  mobility $\mu \simeq 3\text{-}5\ 10^5\ cm^2V^{-1}s^{-1}$
  insulating bulk

Summary of Part I

Versatility of HgTe platform

- several topological systems possible:
  2D/3D TIs, Weyl semi-metal, QAH/QH/QSH + core-shell NW

- high quality samples:
  mobility $\mu \simeq$ 3-5 10^5 $cm^2V^{-1}s^{-1}$
  insulating bulk

Ready for topological quantum computing ?

- reproducible results & easily scalable (large MBE-grown layers)

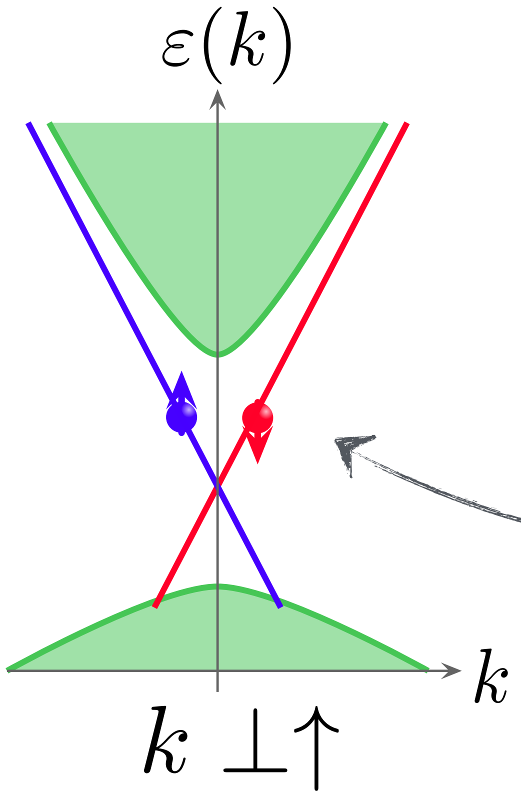
- robust topological states
  ↳ topological protection (QSH) ? band engineering?

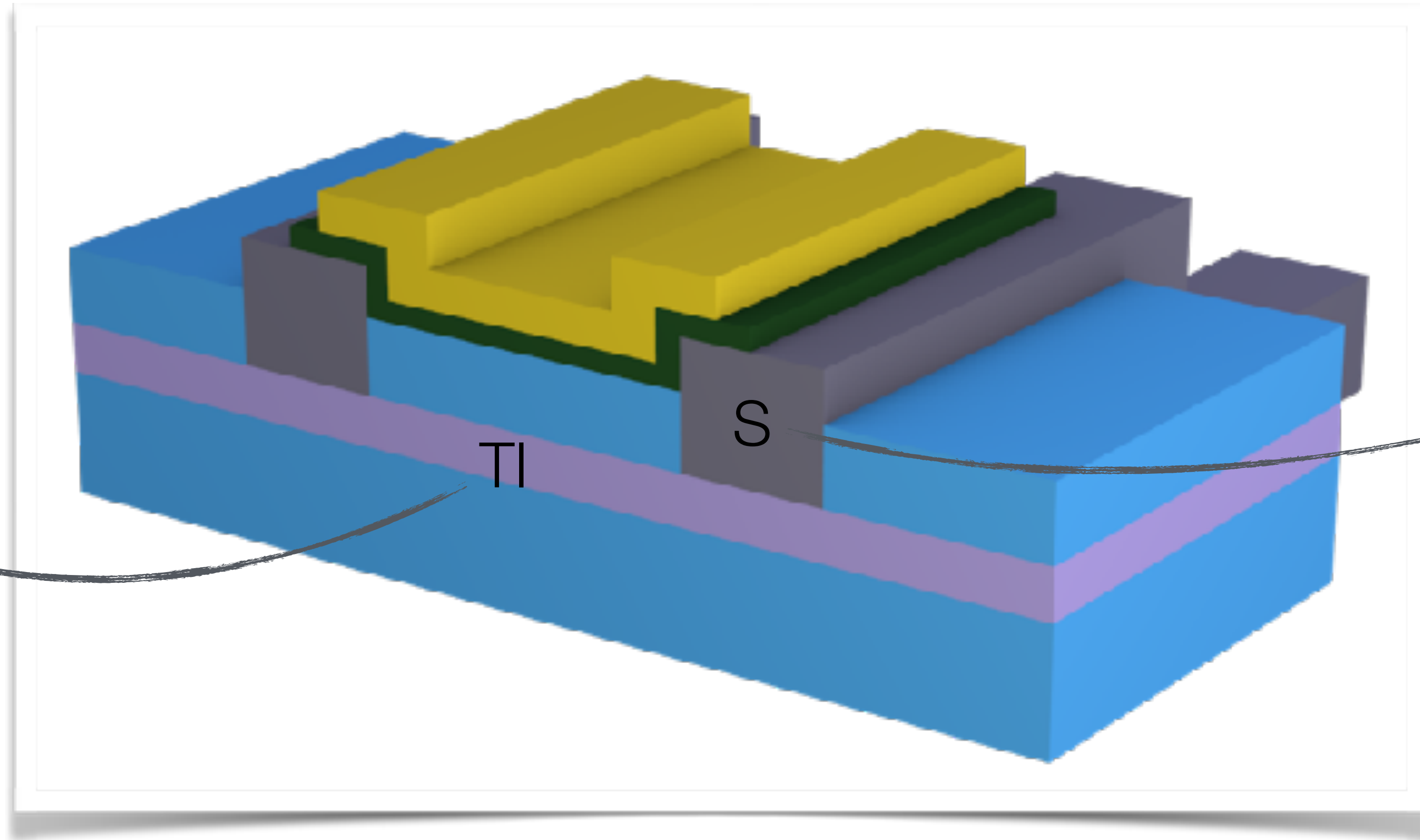
- complex systems with coexisting topological phases :
  ↳ Weyl semi-metal + TSS

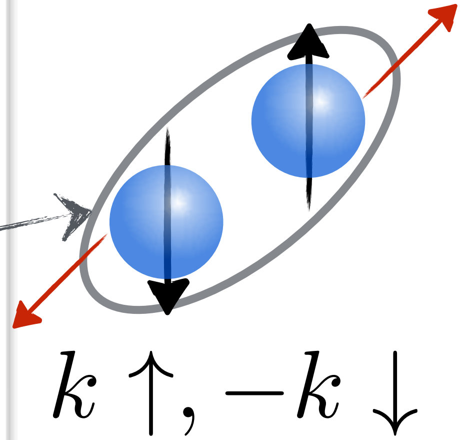
Part II - Superconductivity in HgTe

A. Foreword on topological superconductivity
B. Physics of a Josephson junction
C. Search for gapless ABS in topological JJs
D. Induced superconductivity in S-N junctions

II-A. Topological superconductivity







Cooper pair of helical Dirac fermions
⇒ helical pairing
⇒ *p*-type correlations

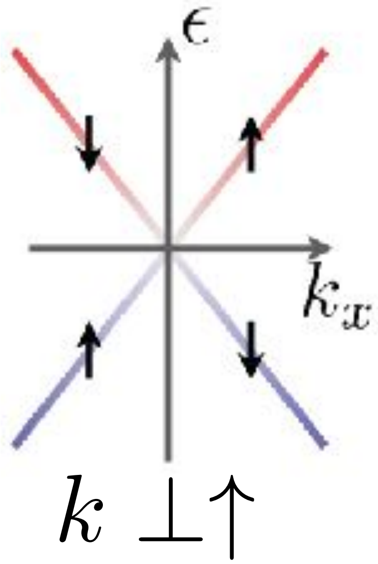
gapless Andreev bound states
⇒ Majoranas

Fu *et al.*, PRB **79**, 161408 (2009)

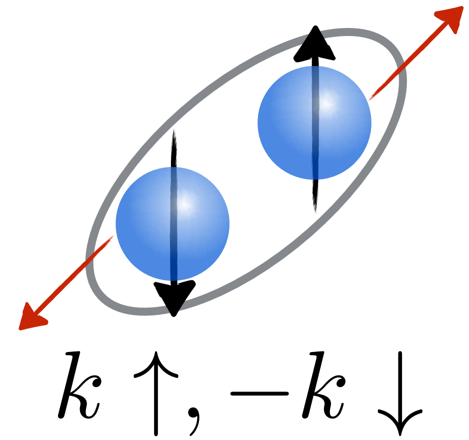
spin-orbit coupling
⇒ φ_0-junctions

Dolcini *et al.*, PRB **92**, 035428 (2015)

II-A. Majoranas and superconductivity



+



=

Cooper pair of helical Dirac fermions

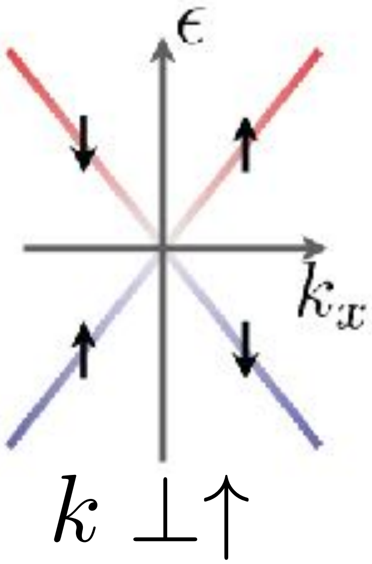
⇒ helical pairing

⇒ *p*-type correlations

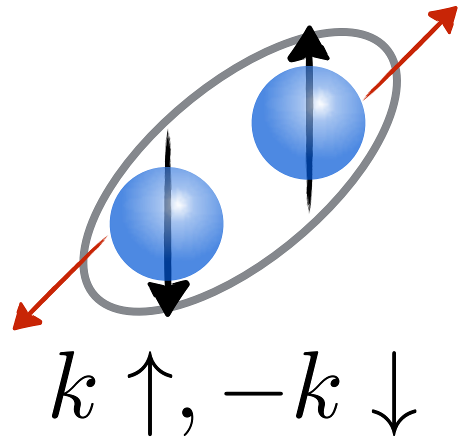
J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012)
C.W.J. Beenakker, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013)

V. Mourik *et al.*, Science **336**, 1003 (2012)
…
S. M. Albrecht *et al.*, Nature **531**, 206 (2016)

II-A. Majoranas and superconductivity



$+$



$=$

Cooper pair of helical Dirac fermions

⇒ helical pairing

⇒ *p*-type correlations

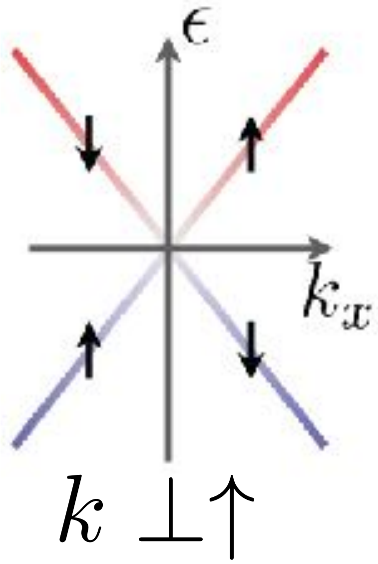
$\dagger$ $=$

$= \frac{1}{2}(c + c^\dagger)$

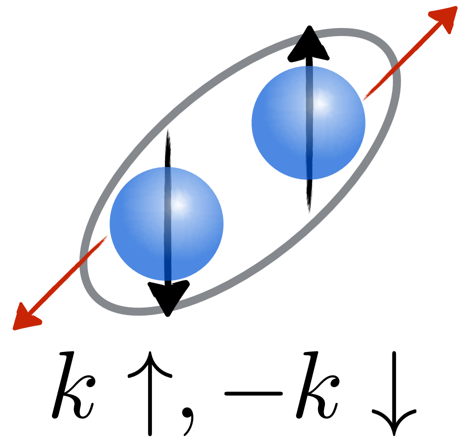
J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012)
C.W.J. Beenakker, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013)

V. Mourik *et al.*, Science **336**, 1003 (2012)
…
S. M. Albrecht *et al.*, Nature **531**, 206 (2016)

II-A. Majoranas and superconductivity



+



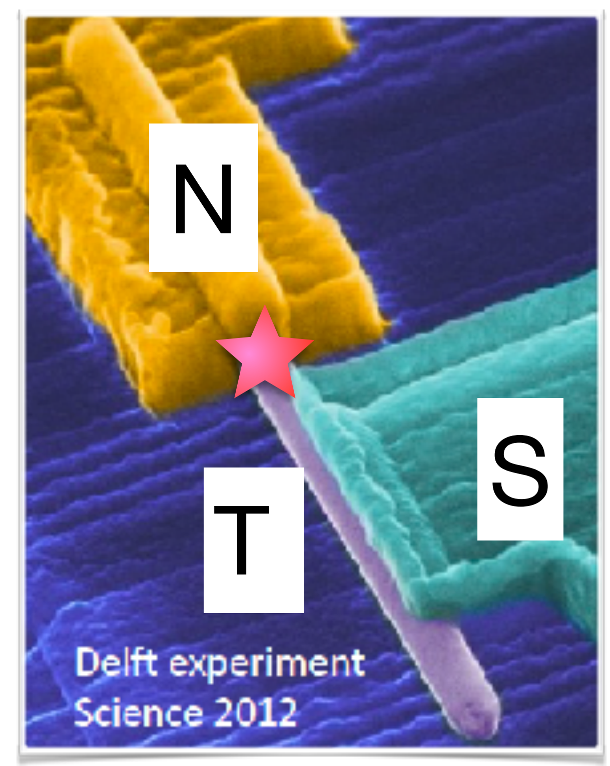
=

Cooper pair of helical Dirac fermions
⇒ helical pairing
⇒ *p*-type correlations


† =



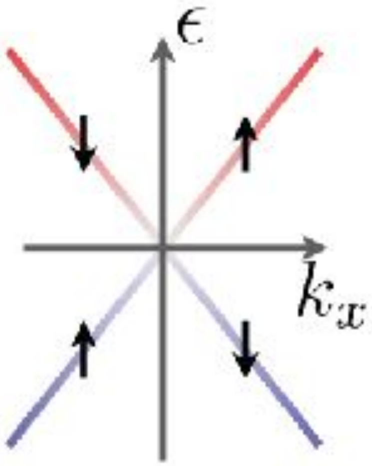
$= \frac{1}{2}(c + c^\dagger)$



J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012)
C.W.J. Beenakker, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013)

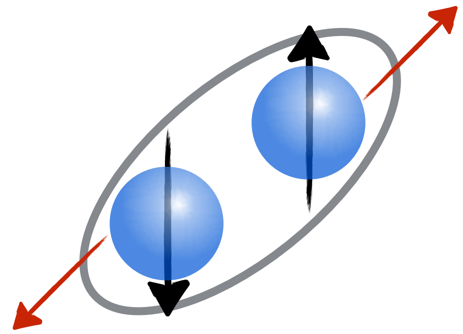
V. Mourik *et al.*, Science **336**, 1003 (2012)
…
S. M. Albrecht *et al.*, Nature **531**, 206 (2016)

II-A. Majoranas and superconductivity



$k \perp \uparrow$

$+$



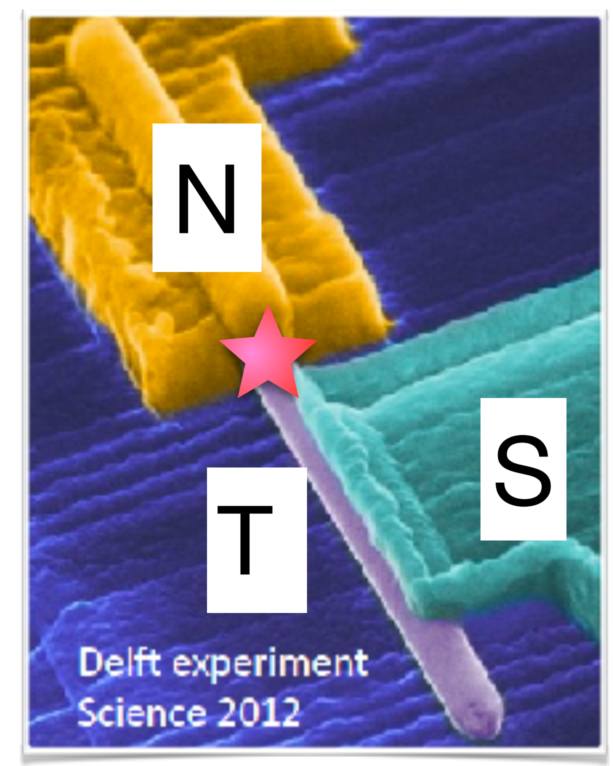
$k\uparrow, -k\downarrow$

$=$

Cooper pair of helical Dirac fermions
⇒ helical pairing
⇒ *p*-type correlations

$=$

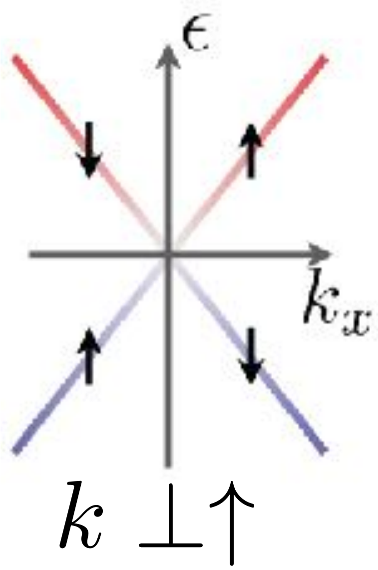
Sayonara fermions ?



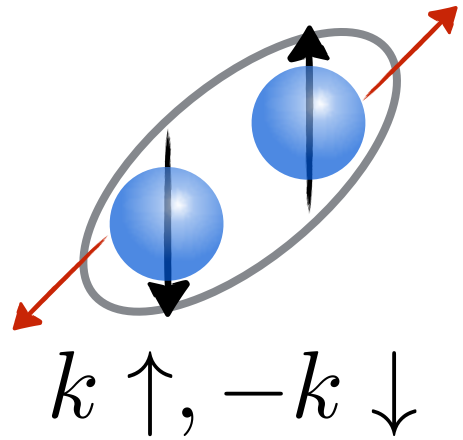
J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012)
C.W.J. Beenakker, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013)

V. Mourik *et al.*, Science **336**, 1003 (2012)
…
S. M. Albrecht *et al.*, Nature **531**, 206 (2016)

II-A. Majoranas and superconductivity

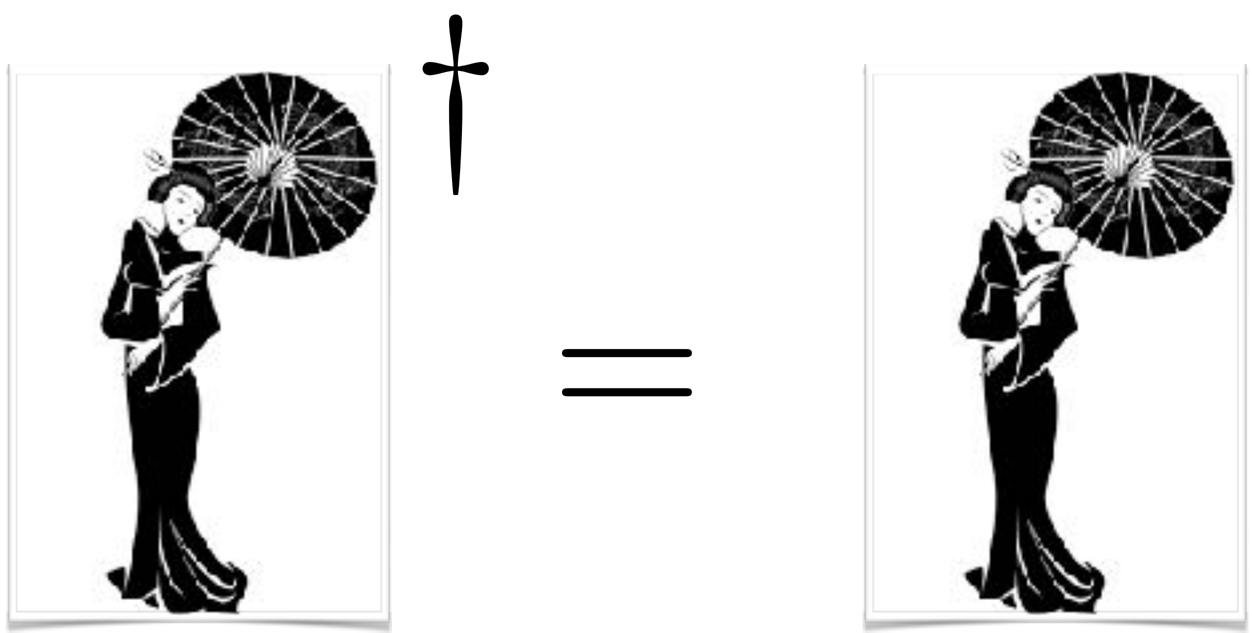


+

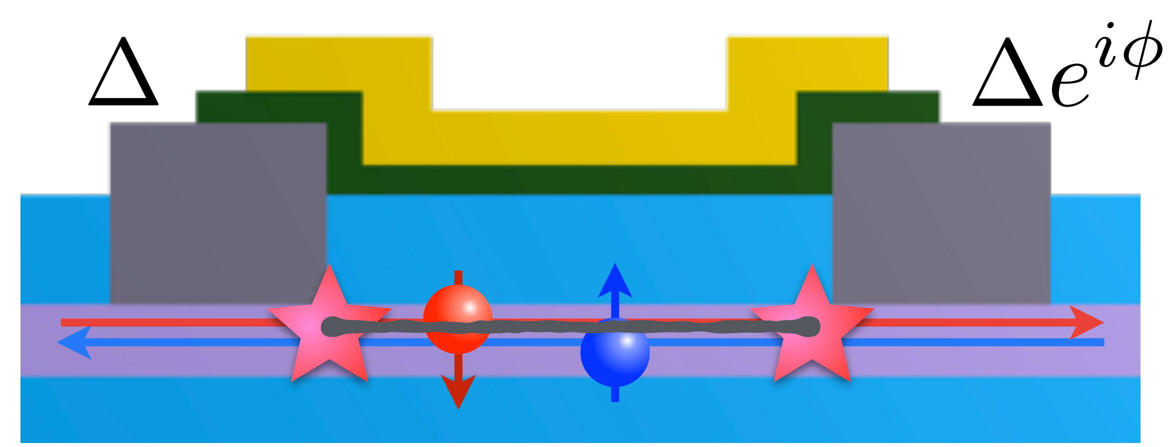


=

Cooper pair of helical Dirac fermions
⇒ helical pairing
⇒ *p*-type correlations

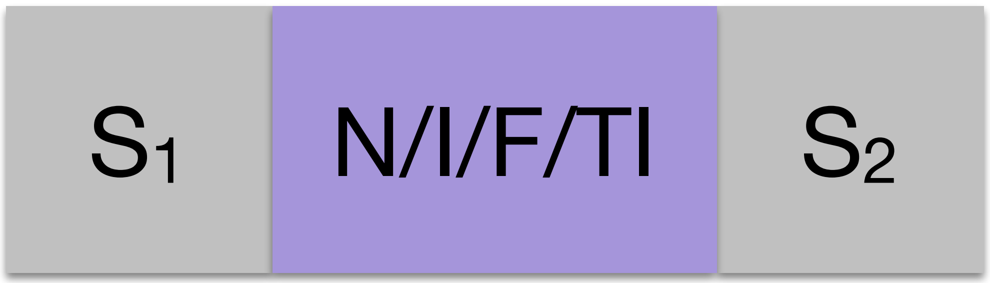


Sayonara fermions ?



J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012)
C.W.J. Beenakker, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013)

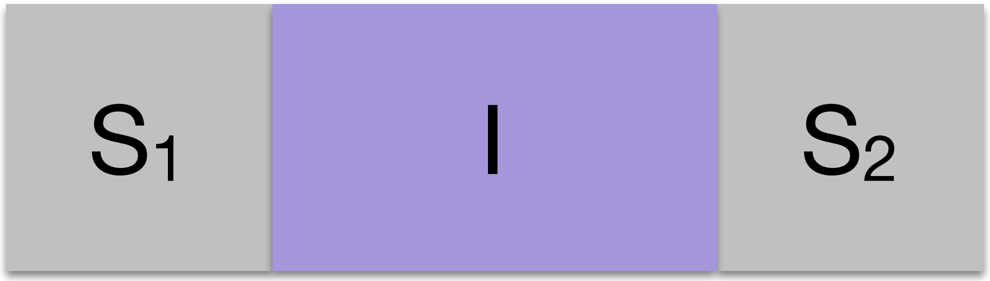
V. Mourik *et al.*, Science **336**, 1003 (2012)
…
S. M. Albrecht *et al.*, Nature **531**, 206 (2016)



Josephson junctions

- 2 superconductors S_j
  $\Psi_j = \sqrt{n_j} e^{i\phi_j}, j = 1, 2$
- coupling through weak link : N/I/F/TI

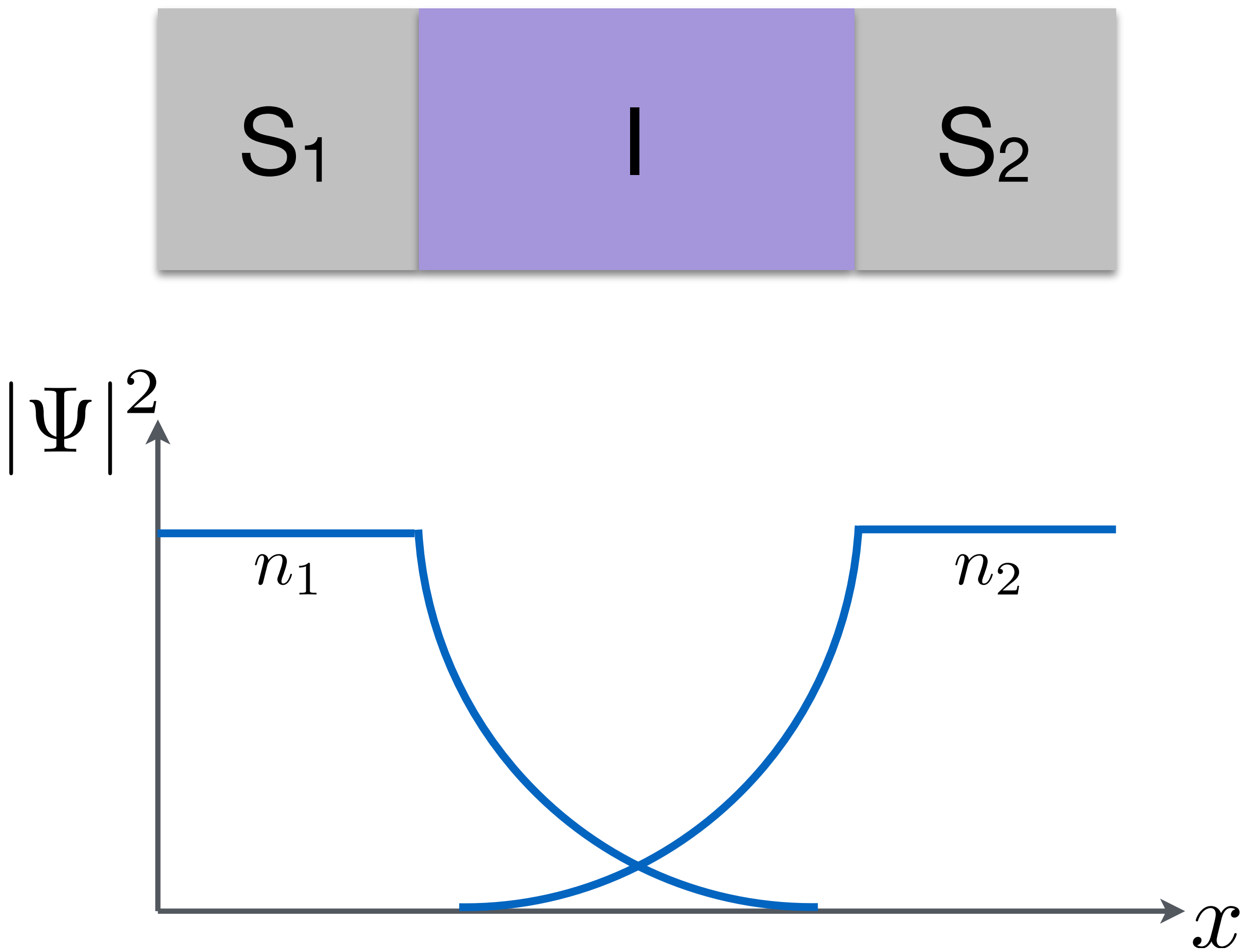
II-B. Josephson junctions



Josephson junctions

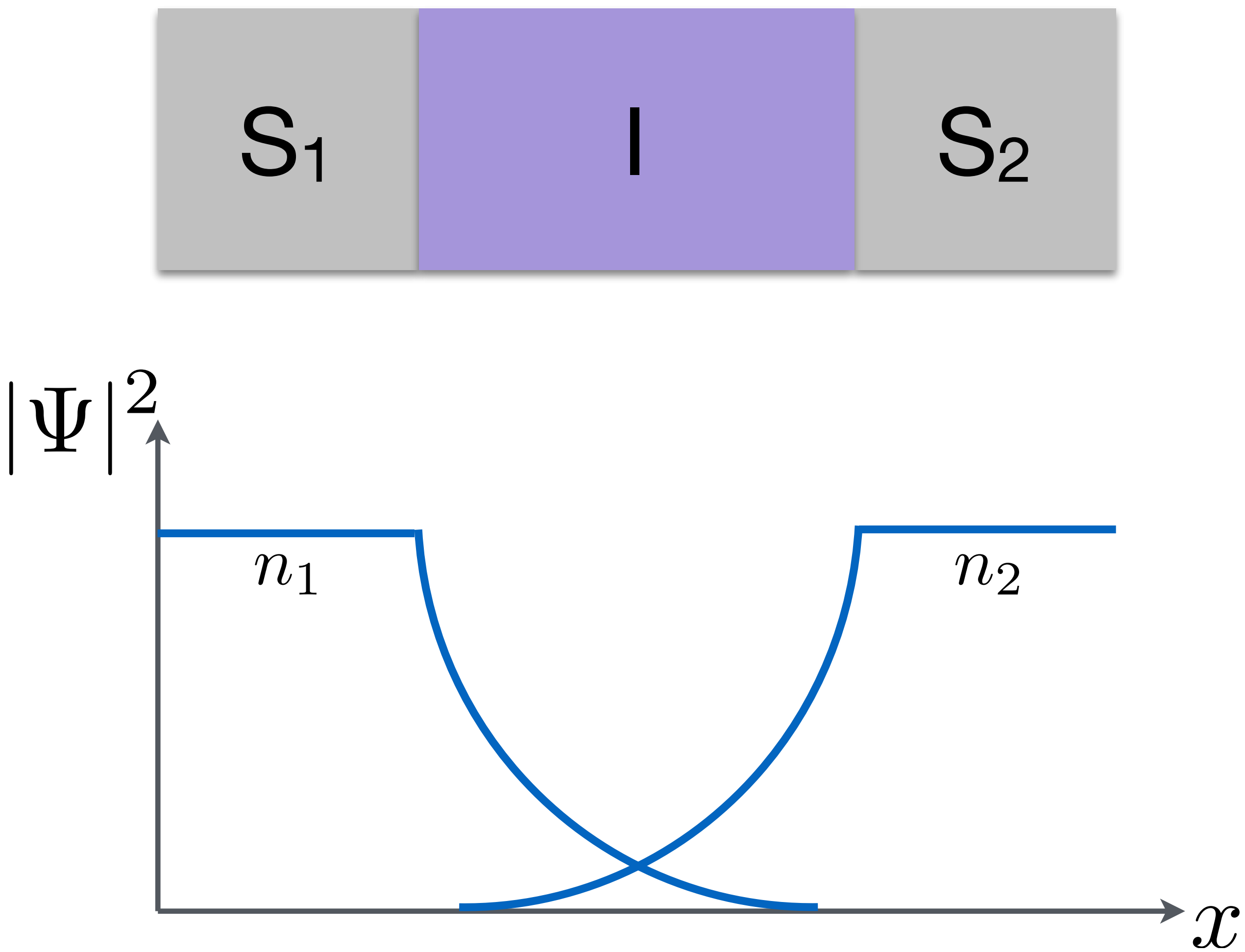
- 2 superconductors S_j
  $\Psi_j = \sqrt{n_j} e^{i\phi_j}, j = 1, 2$
- coupling through weak link : N/I/F/TI

II-B. Josephson junctions



Josephson junctions

- 2 superconductors S_j
  $\Psi_j = \sqrt{n_j}e^{i\phi_j}, j = 1, 2$
- coupling through weak link : **N/I/F/TI**



Josephson junctions

- 2 superconductors S_j
  $\Psi_j = \sqrt{n_j} e^{i\phi_j}, j = 1, 2$
- coupling through weak link : **N/I/F/TI**

Josephson equations

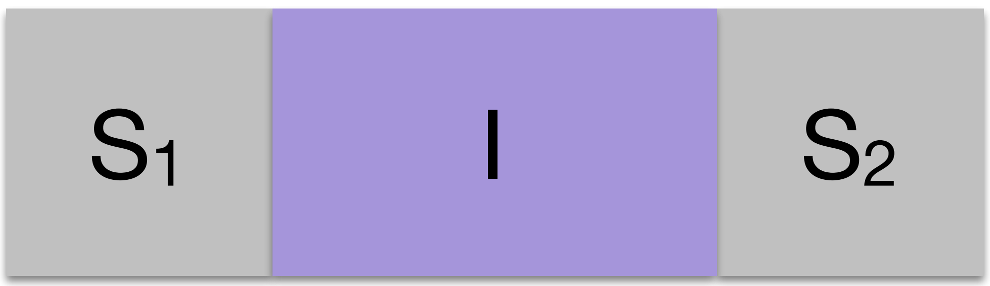
- phase evolution

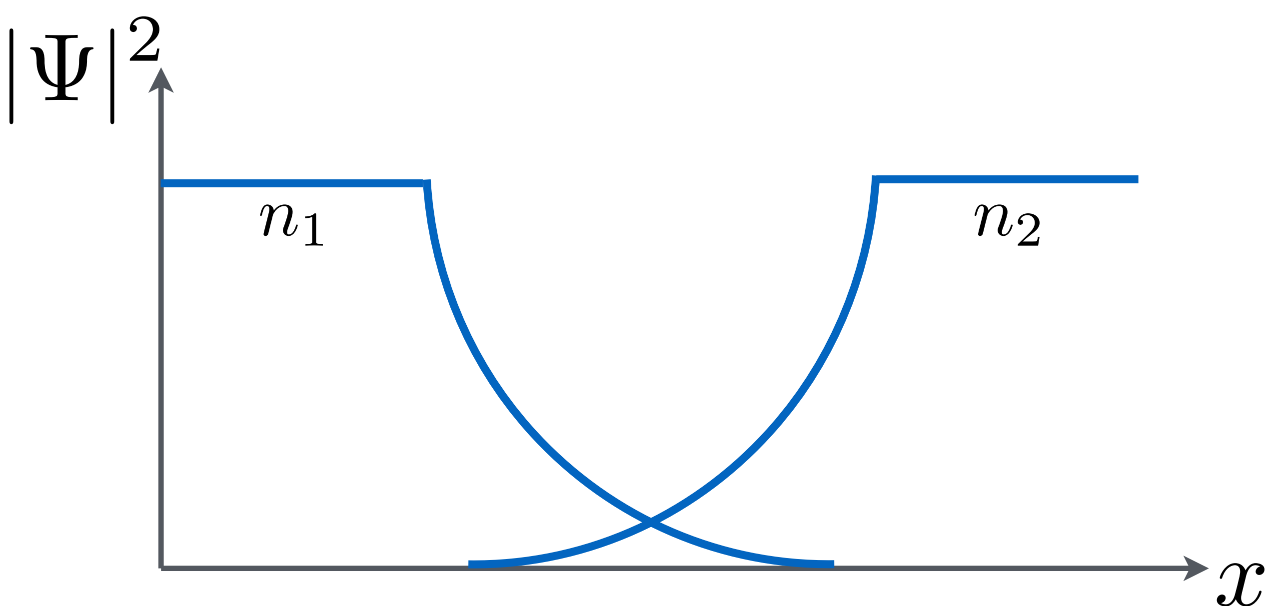
$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

- current-phase relation

$$I_S(\phi) = I_c \sin\phi$$

II-B. Josephson junctions





Josephson junctions

- 2 superconductors S_j

  $$\Psi_j = \sqrt{n_j} e^{i\phi_j}, j = 1, 2$$

- coupling through weak link : **N/I/F/TI**

Josephson equations

- phase evolution

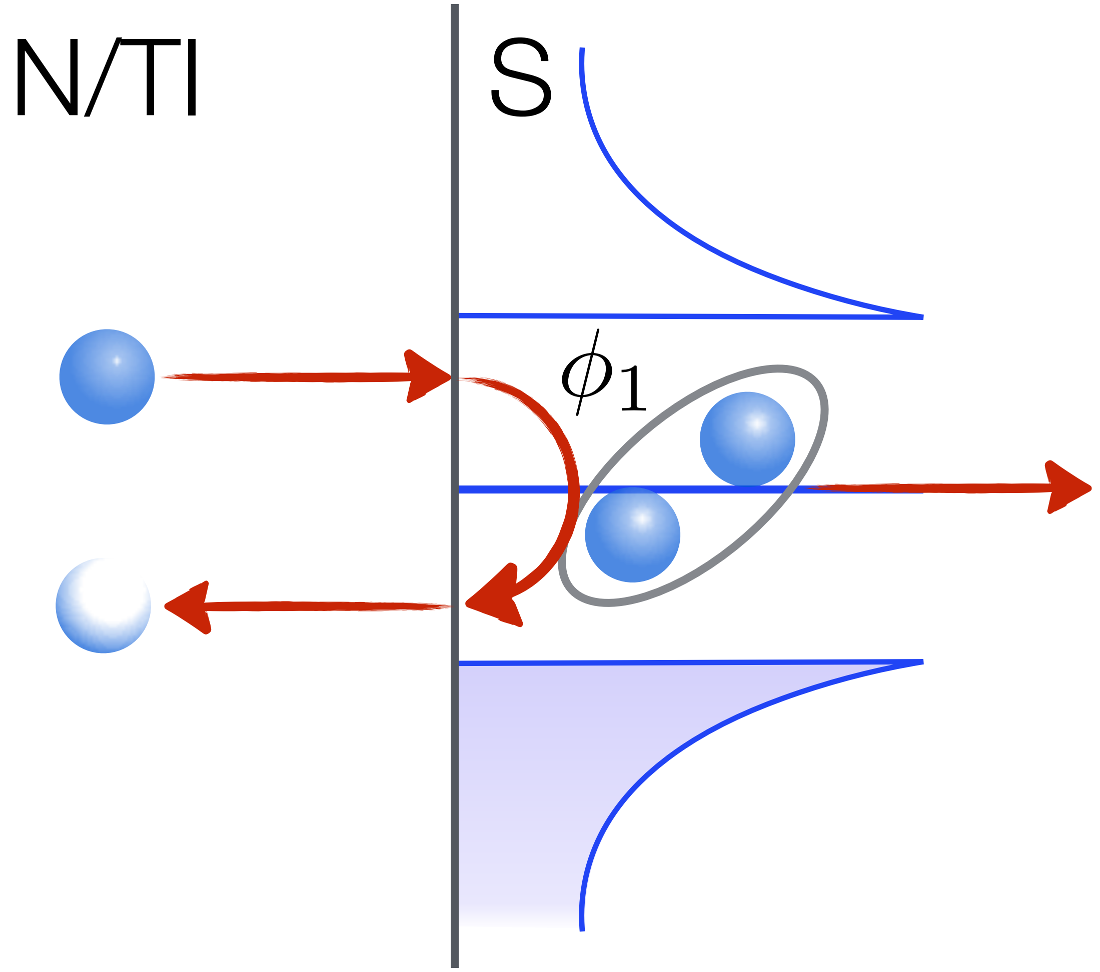
  $$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

- current-phase relation

  $$I_S(\phi) = I_c \sin\phi$$

⇒ supercurrent (I≠0 for V=0) for $I<I_c$ with constant ϕ

finite voltage for $I>I_c$ and ϕ time dependent (Josephson frequency)



Conditions

- two-electron process
- energy/momentum conservation

  ⇒ retro-reflection (exact at E_F)

  $$E_F + \delta E \quad \rightarrow \quad E_F - \delta E$$

  $$\mathbf{k}_F + \delta\mathbf{k} \quad \rightarrow \quad \mathbf{k}_F - \delta\mathbf{k}$$

- phase coherence

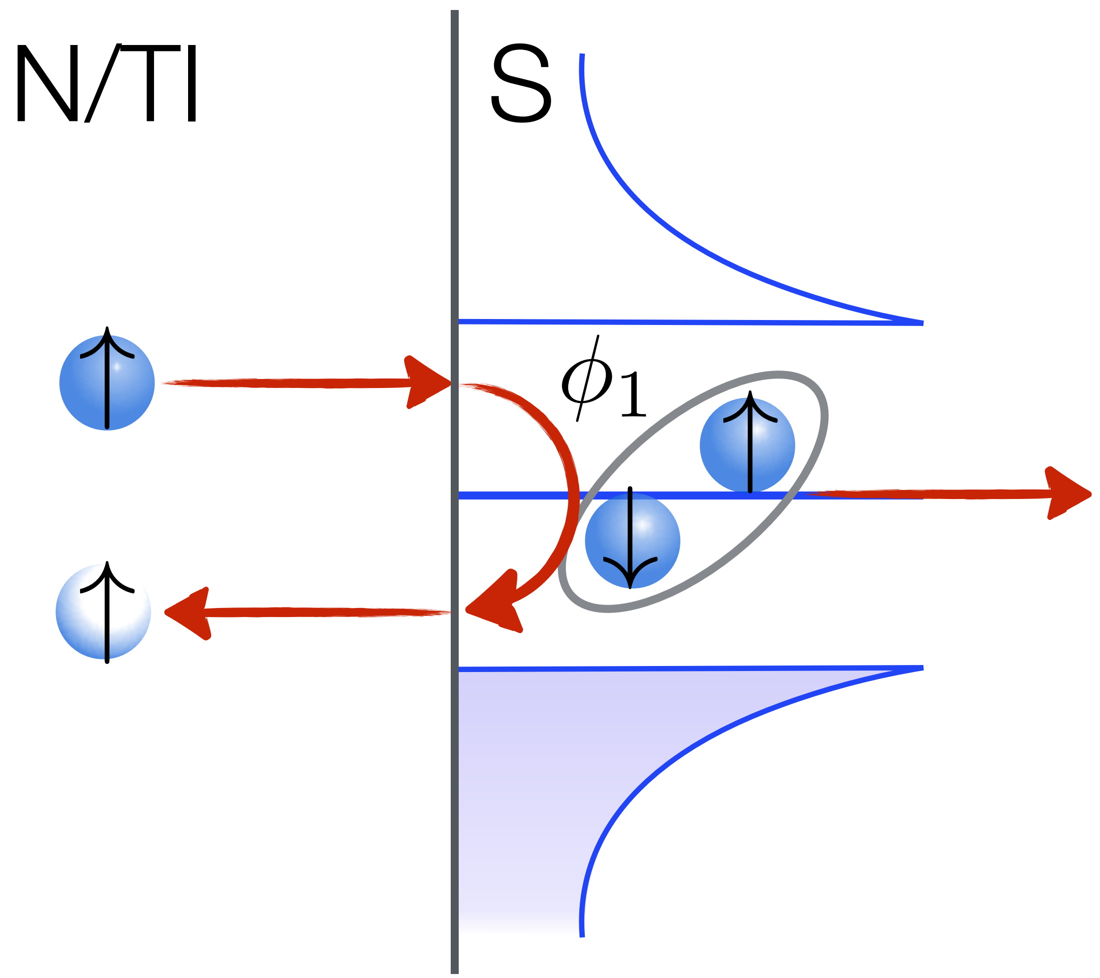
  ⇒ phase shift at interface

  $$\delta\phi = \phi + \arccos\frac{E}{\Delta}$$

- spin conservation

  $$\uparrow \quad \rightarrow \quad \uparrow$$

II-B. Andreev reflections



Conditions

- two-electron process
- energy/momentum conservation

  ⇒ retro-reflection (exact at E_F)

$$E_F + \delta E \quad \rightarrow \quad E_F - \delta E$$

$$\mathbf{k}_F + \delta\mathbf{k} \quad \rightarrow \quad \mathbf{k}_F - \delta\mathbf{k}$$

- phase coherence

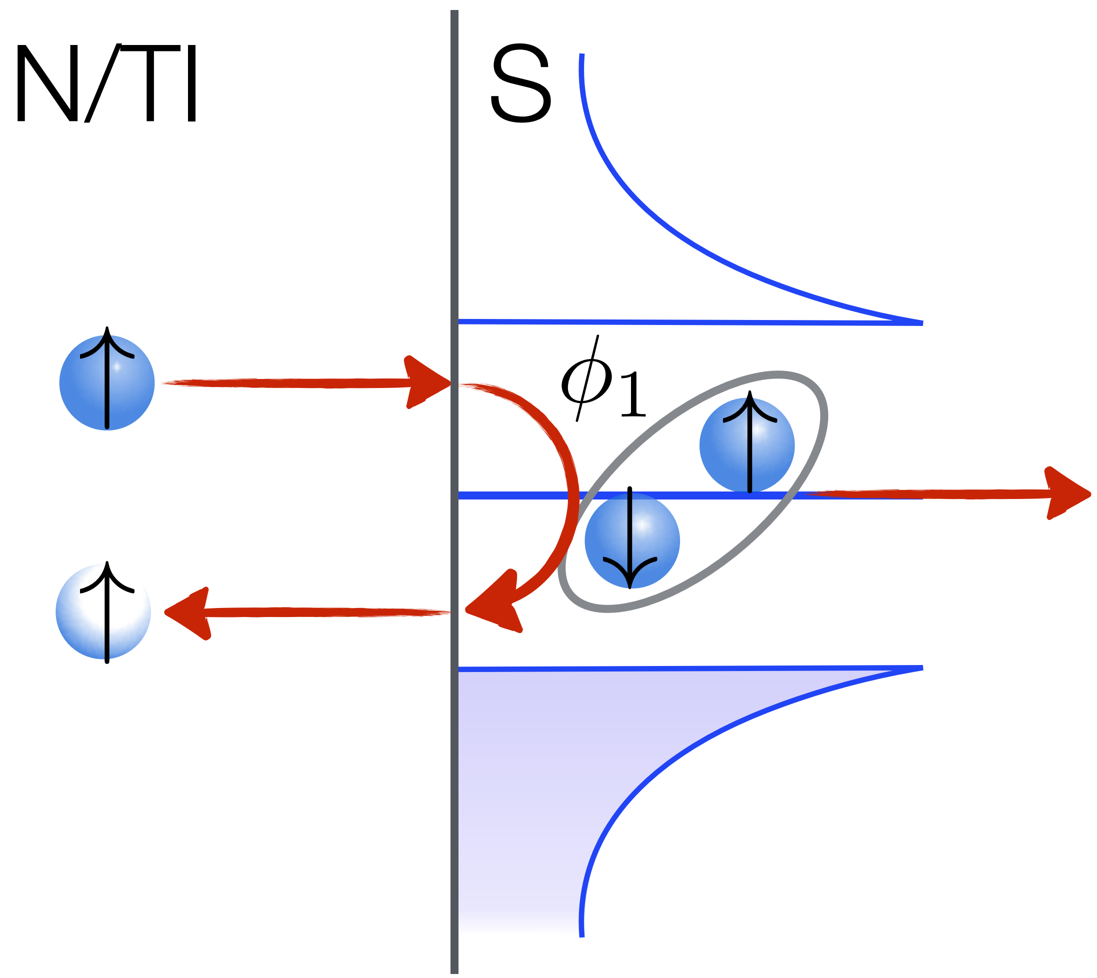
  ⇒ phase shift at interface

$$\delta\phi = \phi + \arccos\frac{E}{\Delta}$$

- spin conservation

$$\uparrow \quad \rightarrow \quad \uparrow$$

II-B. Andreev reflections



Conditions

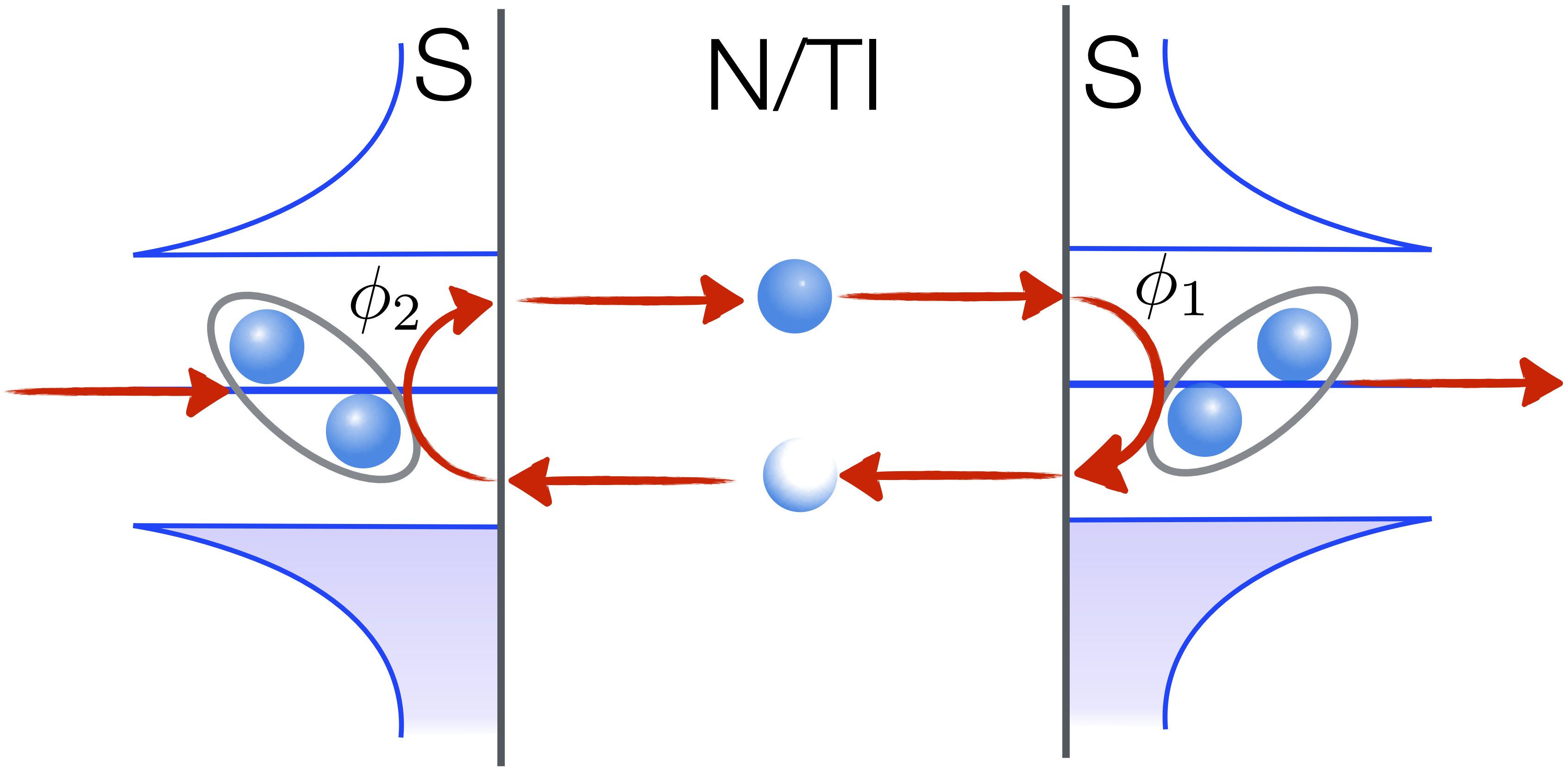
- two-electron process
- energy/momentum conservation
  ⇒ retro-reflection (exact at E_F)
  $$E_F + \delta E \quad \rightarrow \quad E_F - \delta E$$
  $$\mathbf{k}_F + \delta \mathbf{k} \quad \rightarrow \quad \mathbf{k}_F - \delta \mathbf{k}$$
- phase coherence
  ⇒ phase shift at interface
  $$\delta\phi = \phi + \arccos \frac{E}{\Delta}$$
- spin conservation
  $$\uparrow \quad \rightarrow \quad \uparrow$$

® normal reflections if imperfect interface

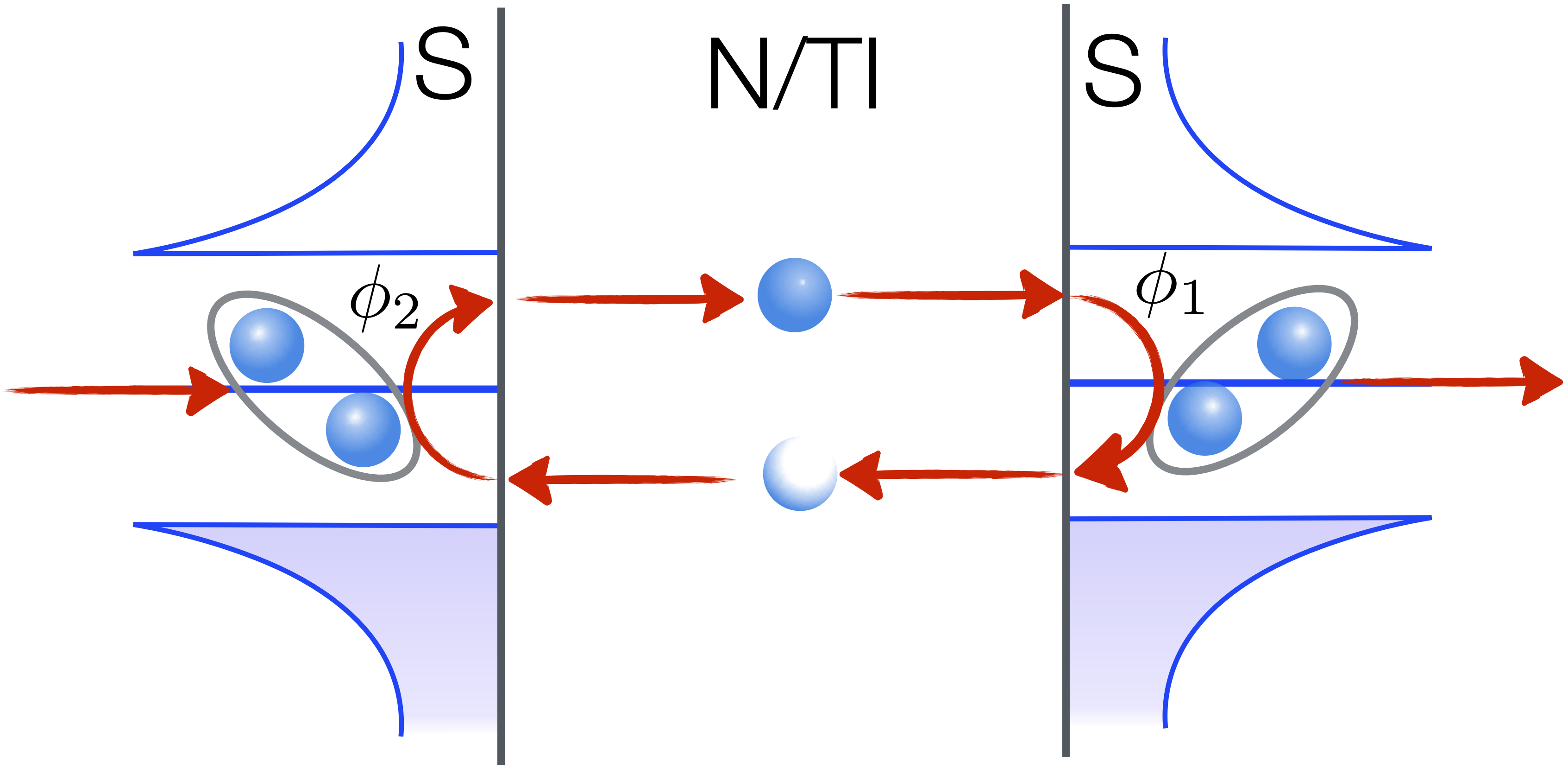
® perfect Andreev reflections in QSH edge states — P. Adroguer *et al.*, PRB **82**, 81303 (2010)

® specular reflections possible in Dirac materials if $E_F \ll \Delta$ — C. W. J. Beenakker, PRL **97** (2006)

II-B. Andreev bound states



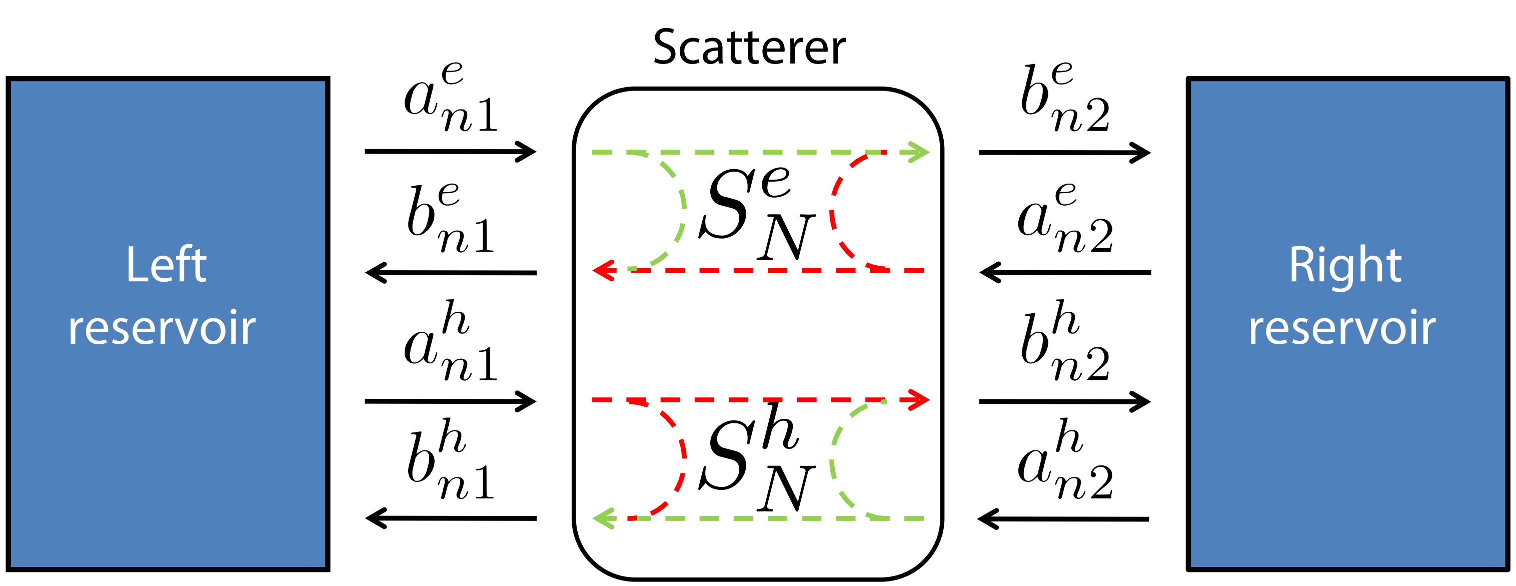
II-B. Andreev bound states



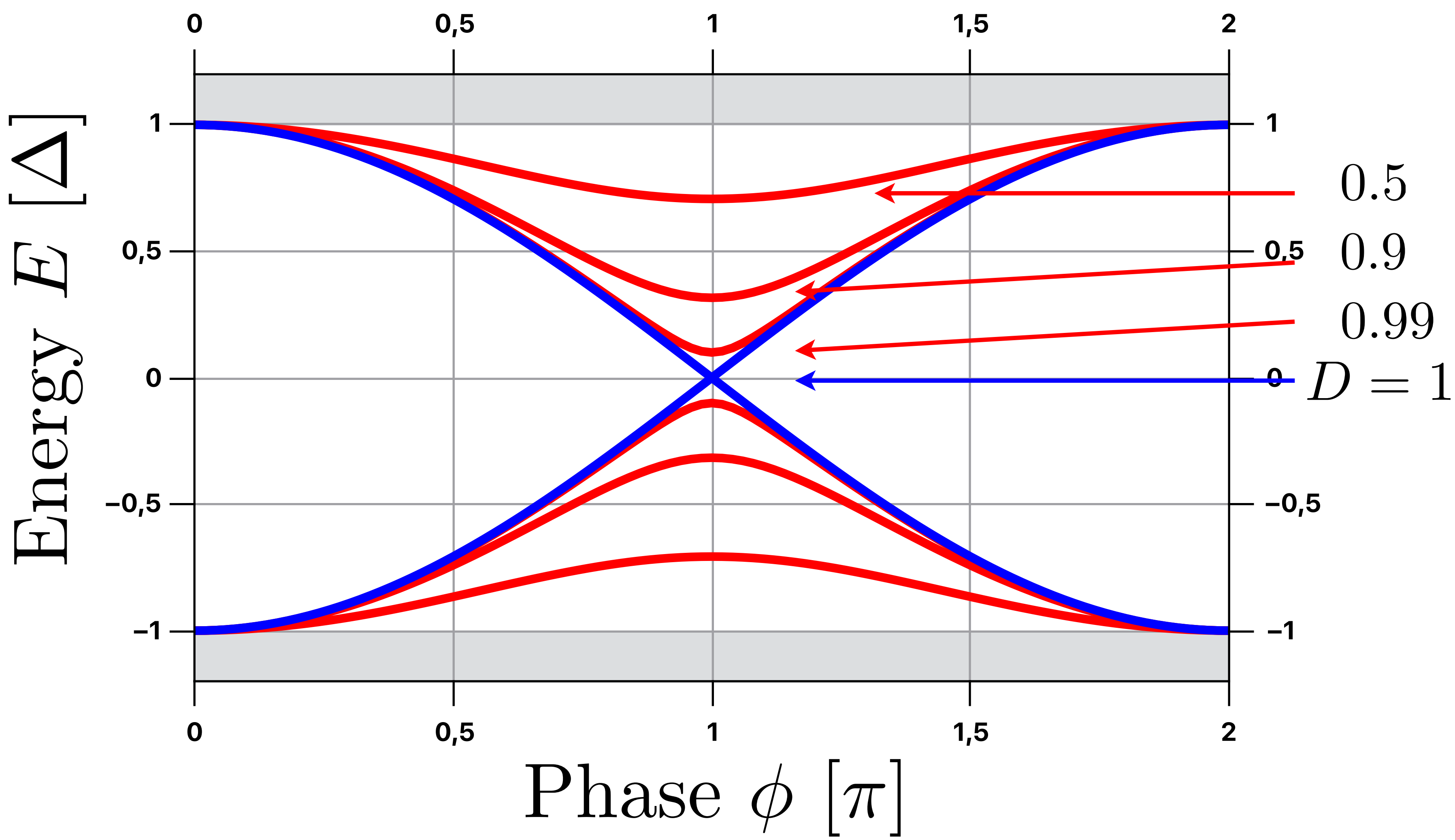
ABS spectrum

- ▹ resonant cavity
- ▹ scattering formalism
  S_A Andreev reflection
  S_N scattering in N region
- ▹ resonance condition

$$\det(I - r_A^{(2)} S_N^h r_A^{(1)} S_N^e) = 0$$



II-B. Conventional Andreev Bound States

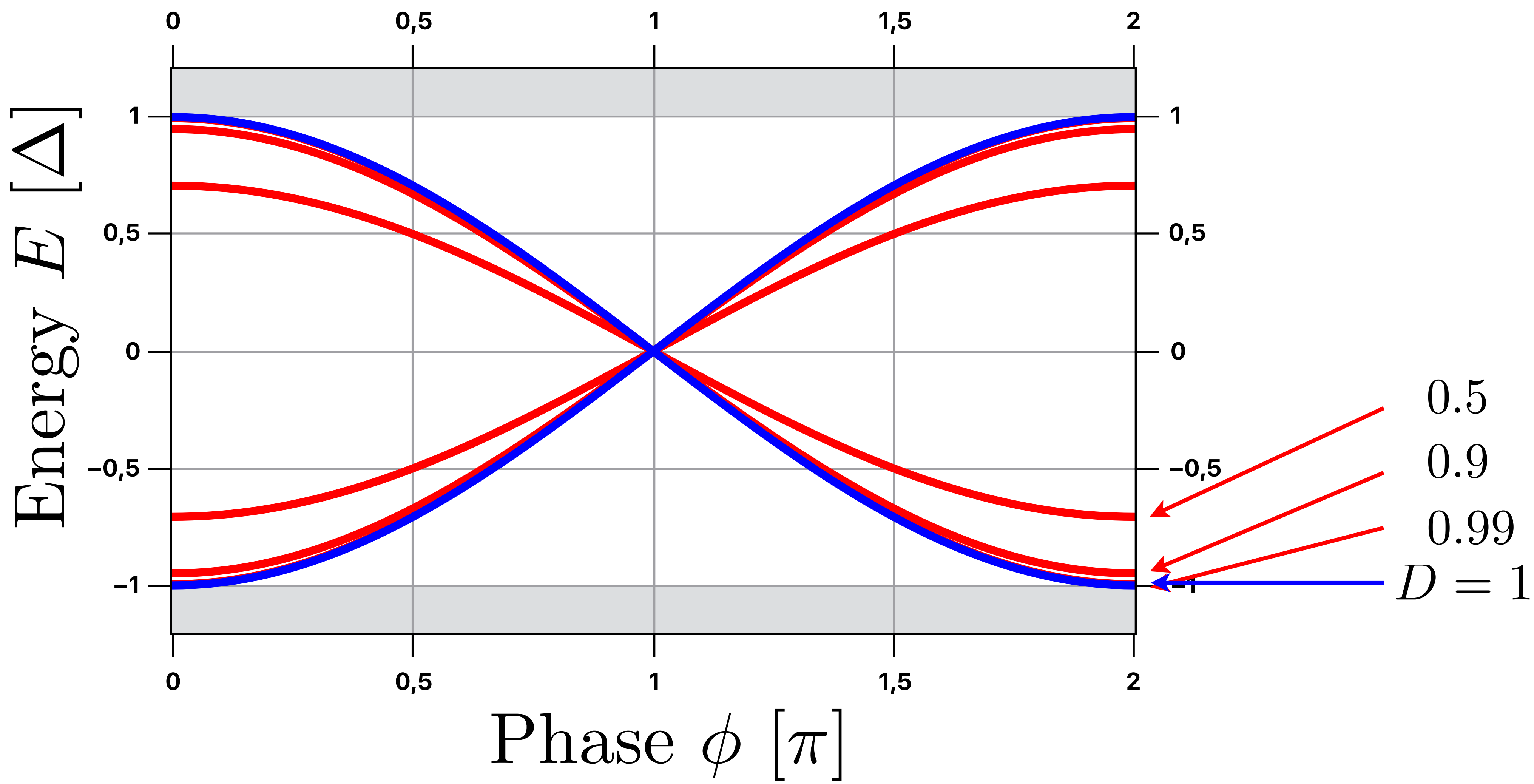


Conventional ABS

- ▹ 1 pair of gapless states
- ▹ spin degeneracy
- ▹ connected to continuum
- ▹ avoided crossing at E=0 for D≠1

$$E(\phi) = \sqrt{1 - D\sin^2\frac{\phi}{2}}$$

II-B. Gapless Andreev Bound States



Topological gapless ABS

- 1 pair of gapless states
- no spin degeneracy
- disconnected from continuum for D≠1
- protected crossing at E=0
- « hybridized Majorana states »

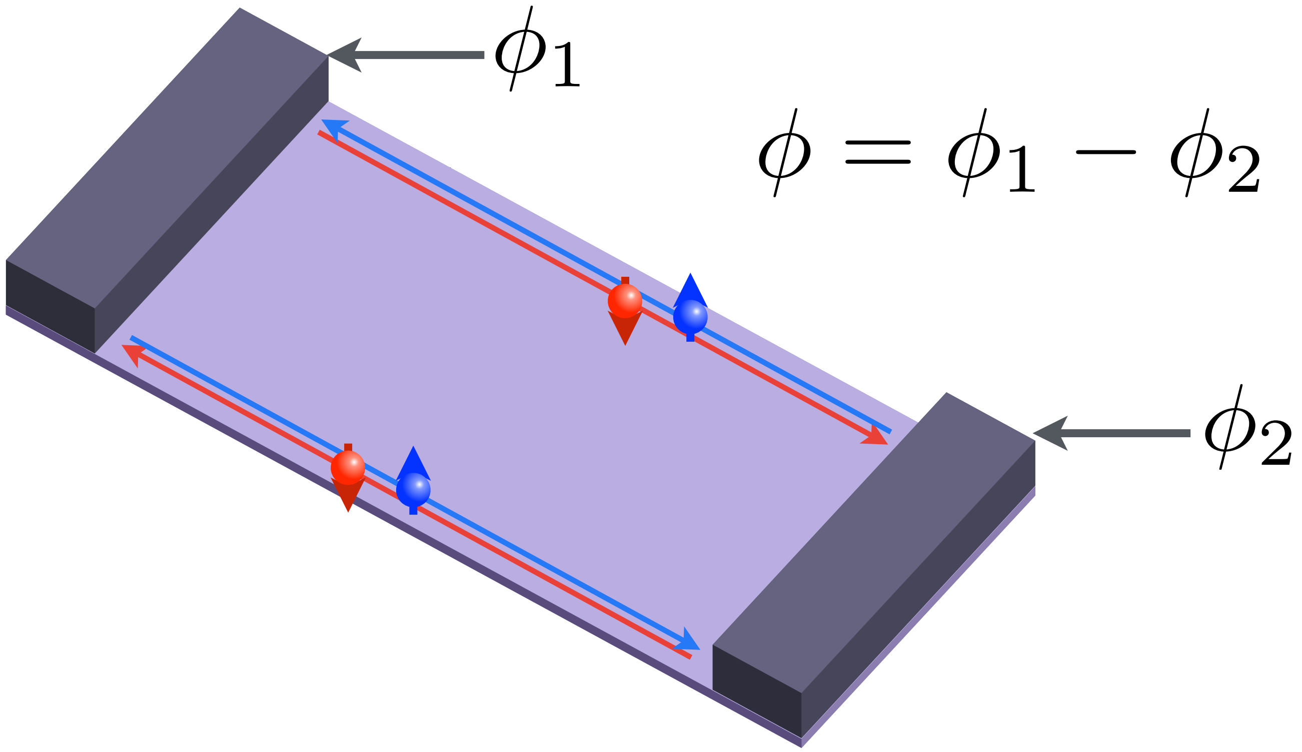
$$E(\phi) = \sqrt{D} \cos \frac{\phi}{2}$$

II-B. (Fractional) Josephson effect

Josephson equations

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

$$I(\phi) = \sum_n \frac{\partial E_n}{\partial \phi}\left(1 - 2f(E_n)\right)$$



II-B. (Fractional) Josephson effect

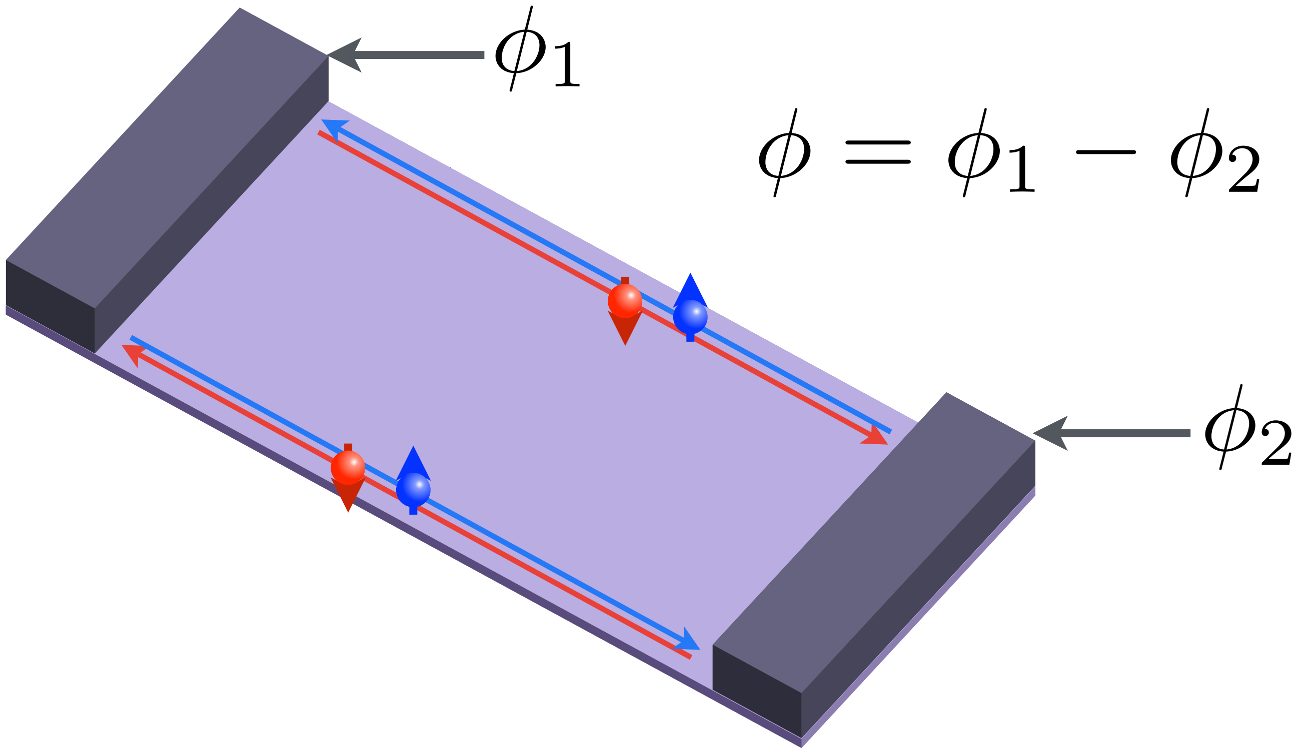
Josephson equations

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

$$I(\phi) = \sum_n \frac{\partial E_n}{\partial \phi}\left(1 - 2f(E_n)\right)$$

$$I_S(\phi) = I_c \sin\phi$$

⇒ Josephson frequency $f_{\mathrm{J}} = \frac{2eV}{\hbar}$



II-B. (Fractional) Josephson effect

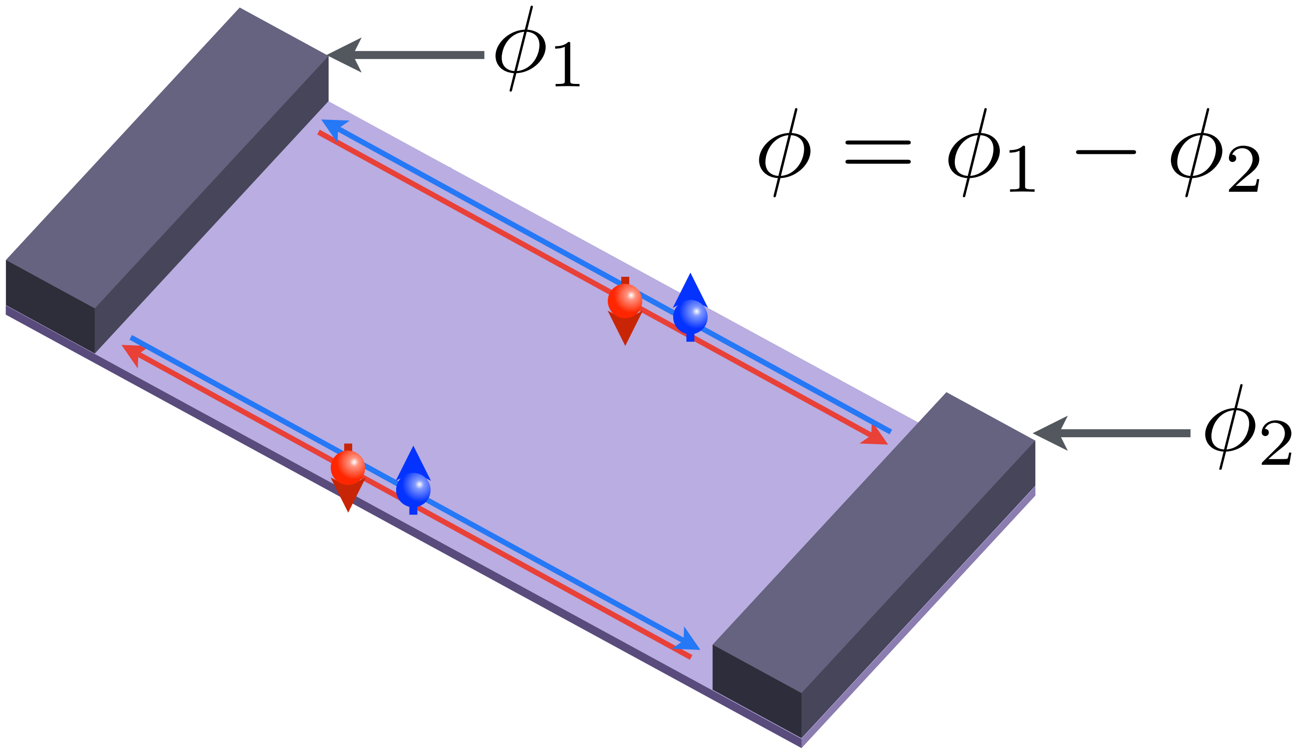
Josephson equations

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

$$I(\phi) = \sum_n \frac{\partial E_n}{\partial \phi}(1 - 2f(E_n))$$

$$I_S(\phi) = I_c \sin\phi$$

⇒ Josephson frequency $f_\mathrm{J} = \frac{2eV}{\hbar}$



Fractional Josephson effect

$$\sin\phi \rightarrow \sin\phi/2$$

$$f_\mathrm{J} \rightarrow f_\mathrm{J}/2$$

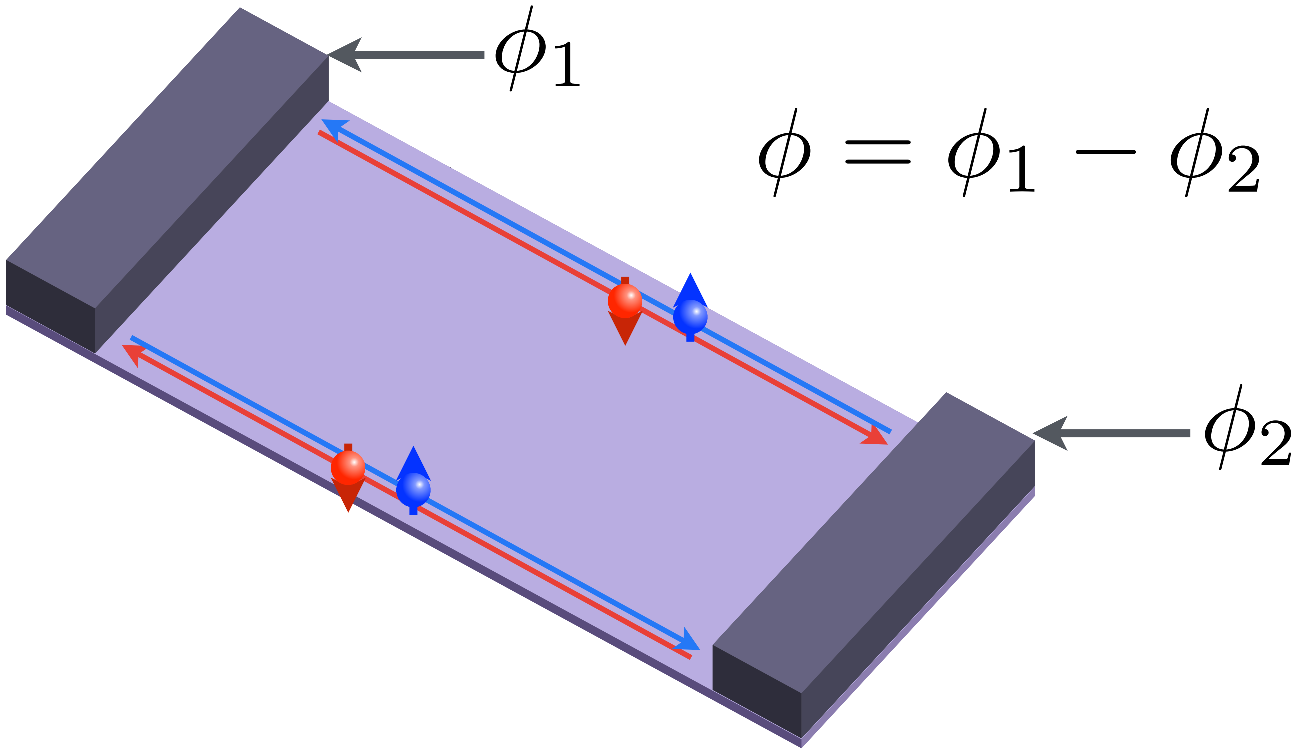
Josephson equations

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

$$I(\phi) = \sum_n \frac{\partial E_n}{\partial \phi}(1 - 2f(E_n))$$

$$I_S(\phi) = I_c \sin\phi$$

⇒ Josephson frequency $f_\mathrm{J} = \frac{2eV}{\hbar}$



Fractional Josephson effect

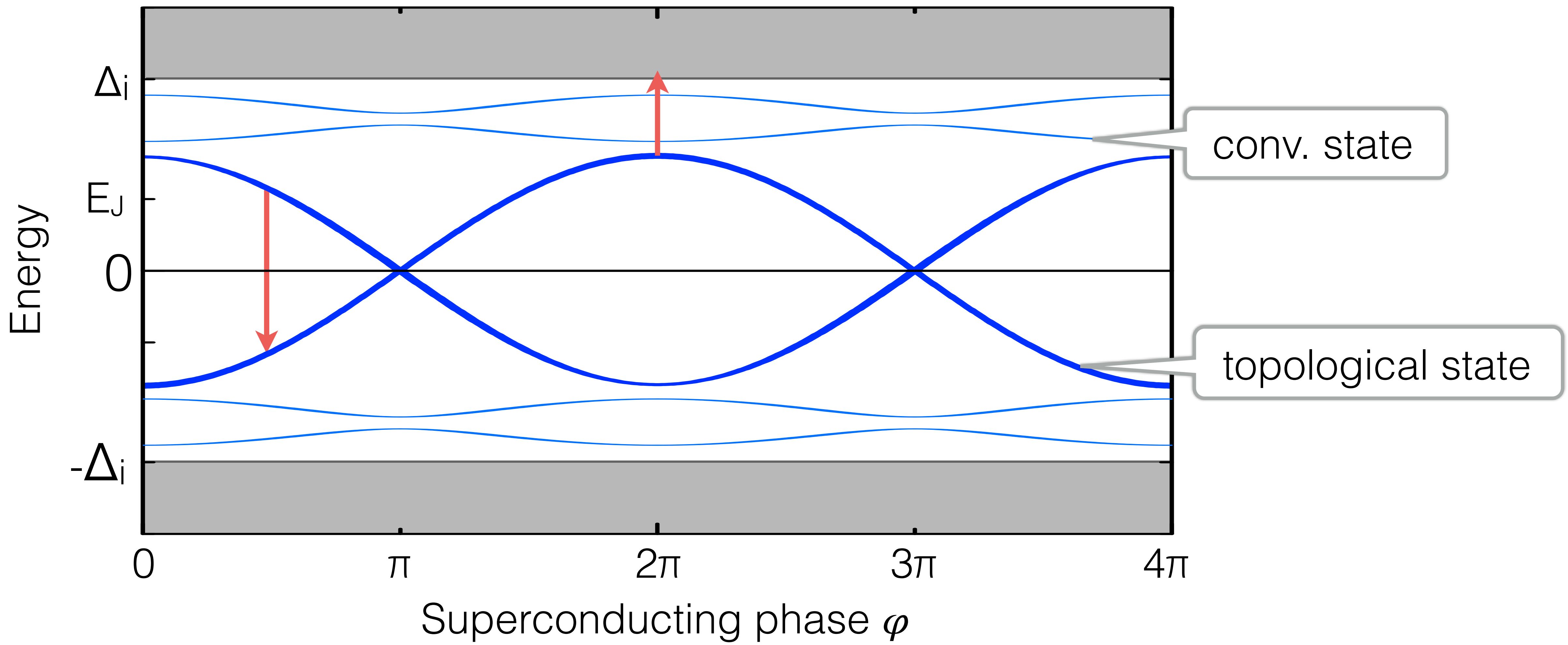
$$\sin\phi \to \sin\phi/2$$

$$f_\mathrm{J} \to f_\mathrm{J}/2$$

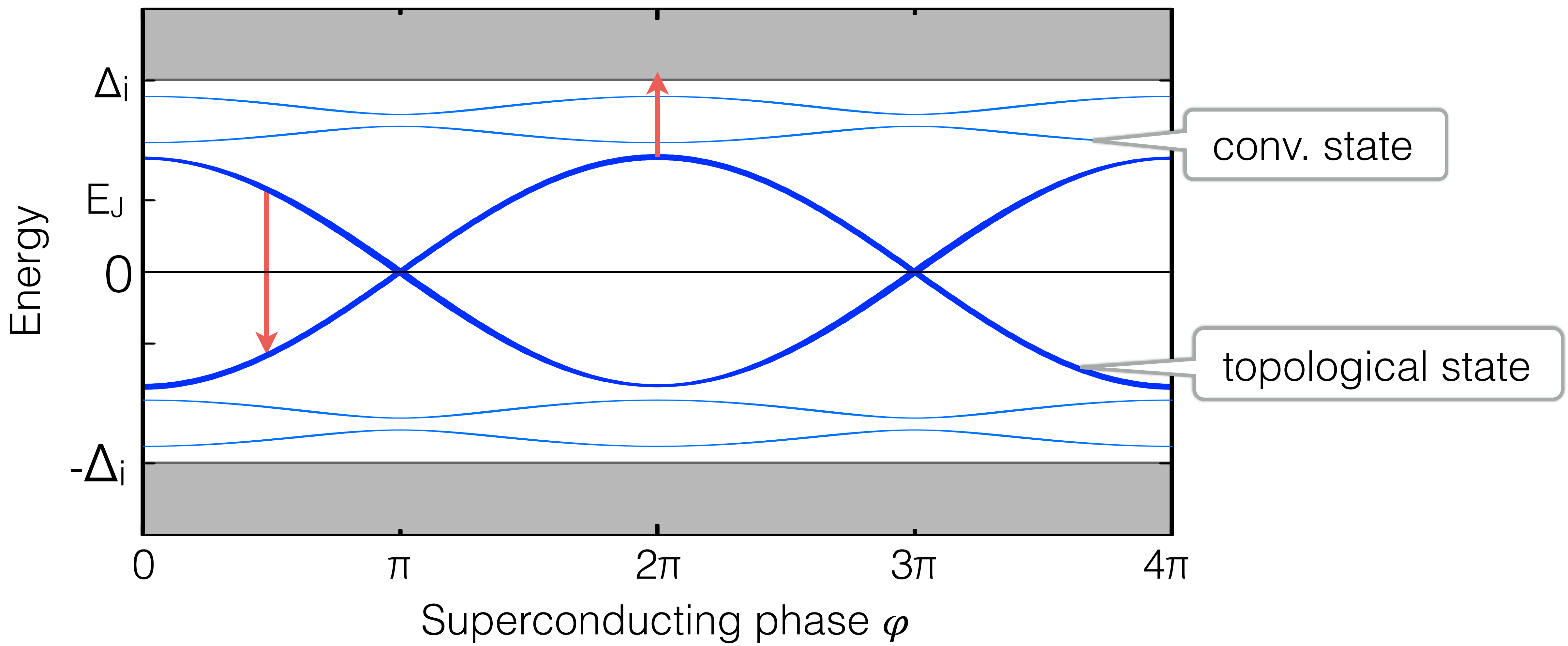
Detection

- ‘listening’ to Josephson emission
- beatings with ac excitation (Shapiro steps)

II-B. Fractional Josephson effect



II-B. Fractional Josephson effect

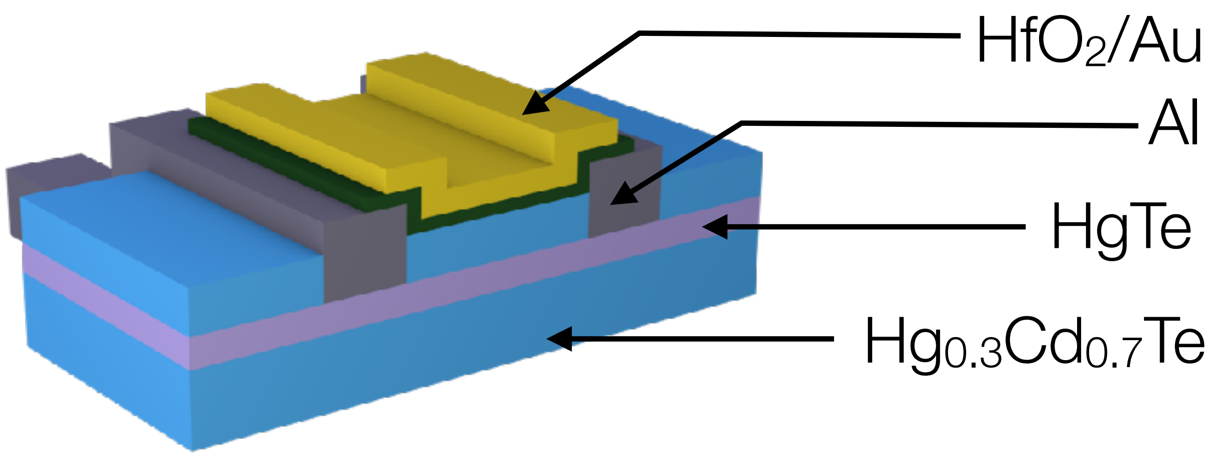


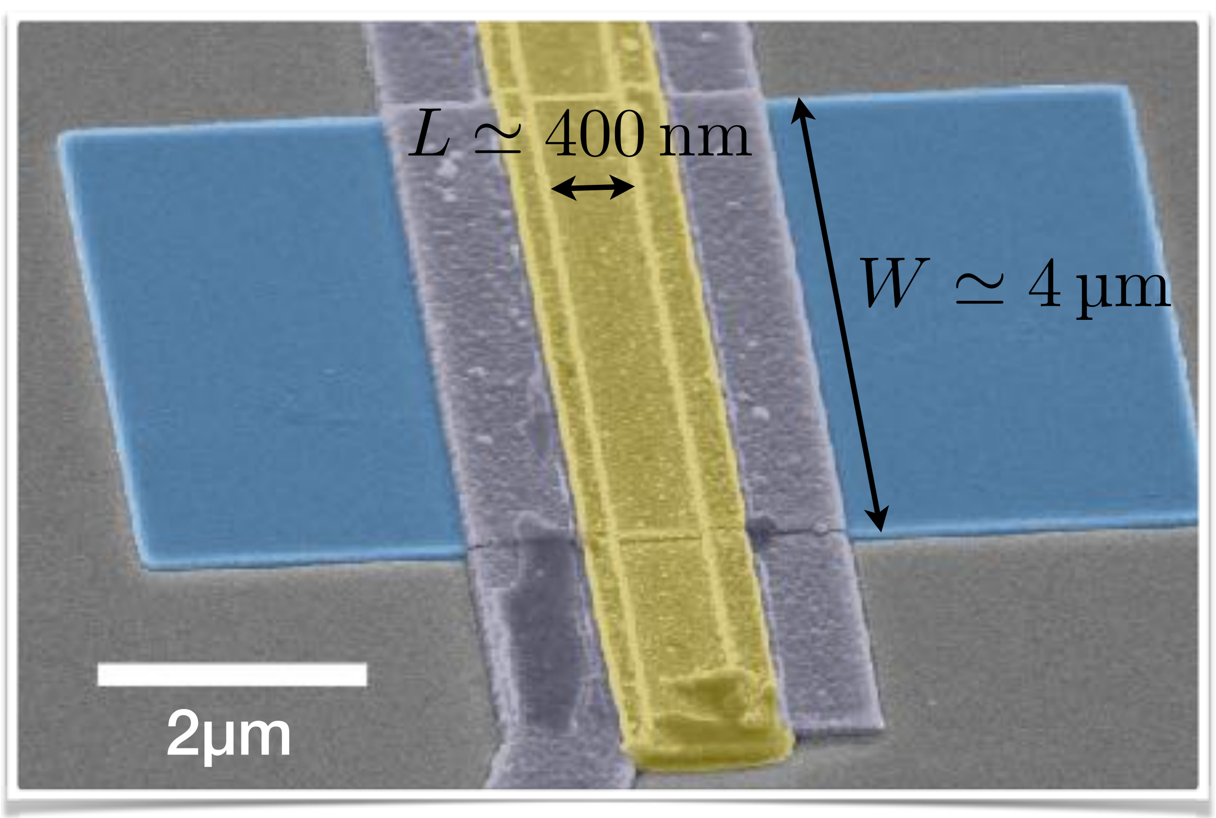
® Many parasitic effects !

- 2π bulk states ⇒ 2π/4π mixture
- finite lifetime/continuum ⇒ 2π-periodicity restored
- interactions ⇒ 8π-periodicity
- Landau-Zener transitions ⇒ 4π-periodicity

Pikulin *et al.*, PRB **86**, 140504 (2012)
Badiane *et al.*, CRP **14**, 840 (2013)
Zhang *et al.*, PRL **113**, 036401 (2014)
Peng *et al.*, PRL **117**, 267001 (2016)
Hui *et al.*, PRB **95**, 014505 (2017)

II-B. Quantum spin Hall junctions



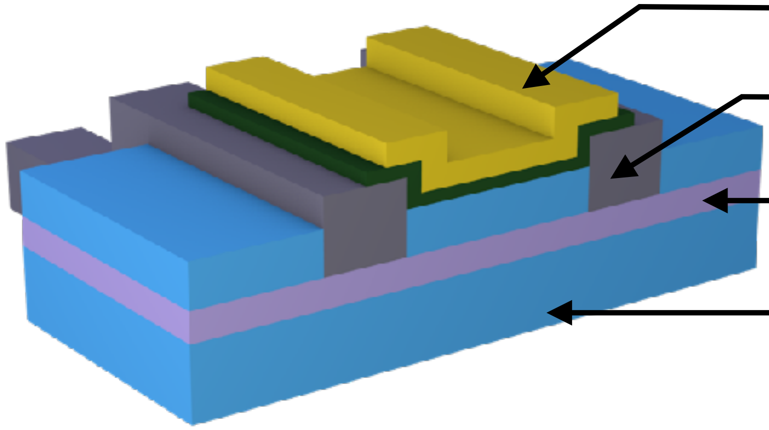


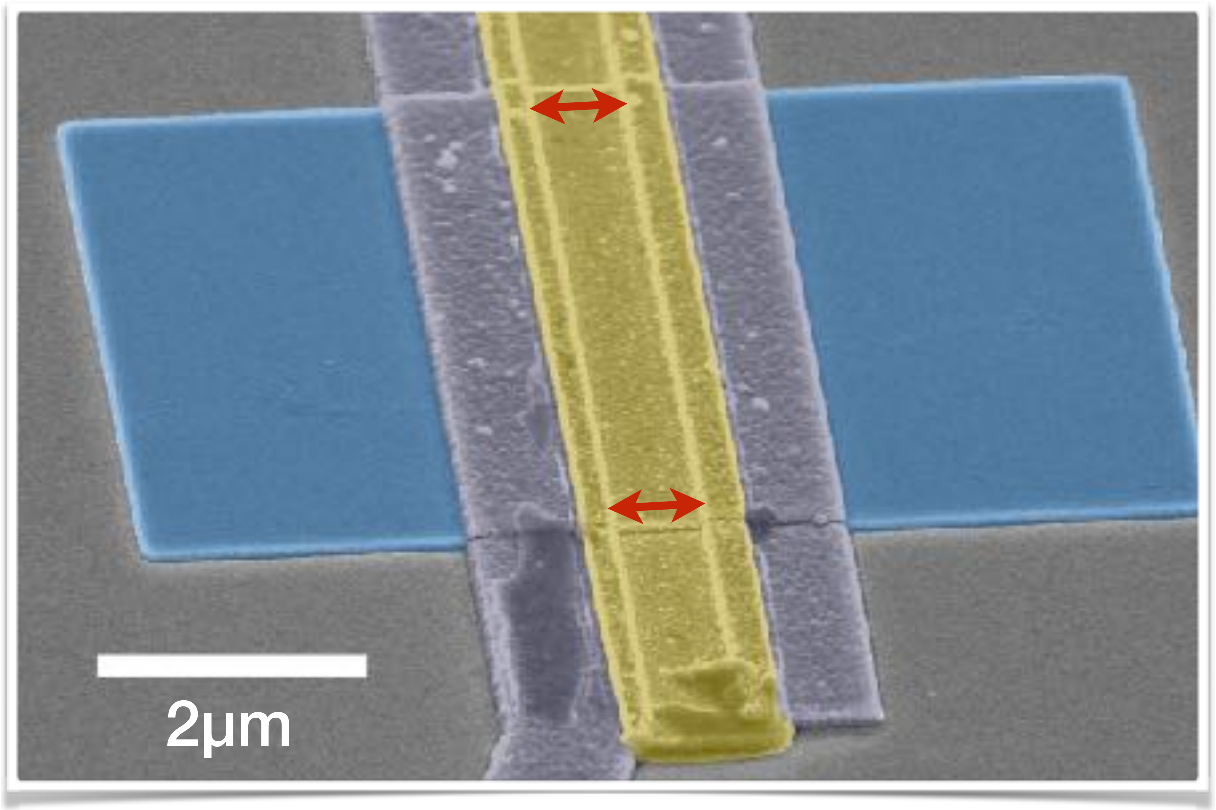
Josephson junctions

- $\mu \simeq 3\ 10^5\ cm^2V^{-1}s^{-1}$
- Al contacts (in situ)
- HfO_2 /Au gate
- no overlap of edge states
- ballistic / intermediate

$$L \ll l \qquad L \lesssim \xi$$

II-B. Quantum spin Hall junctions



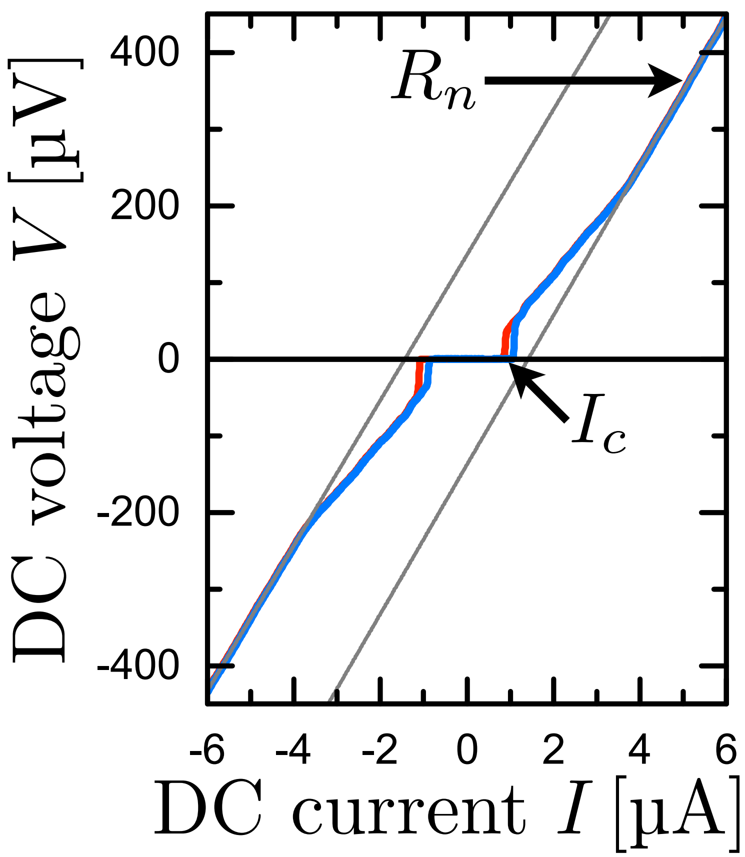


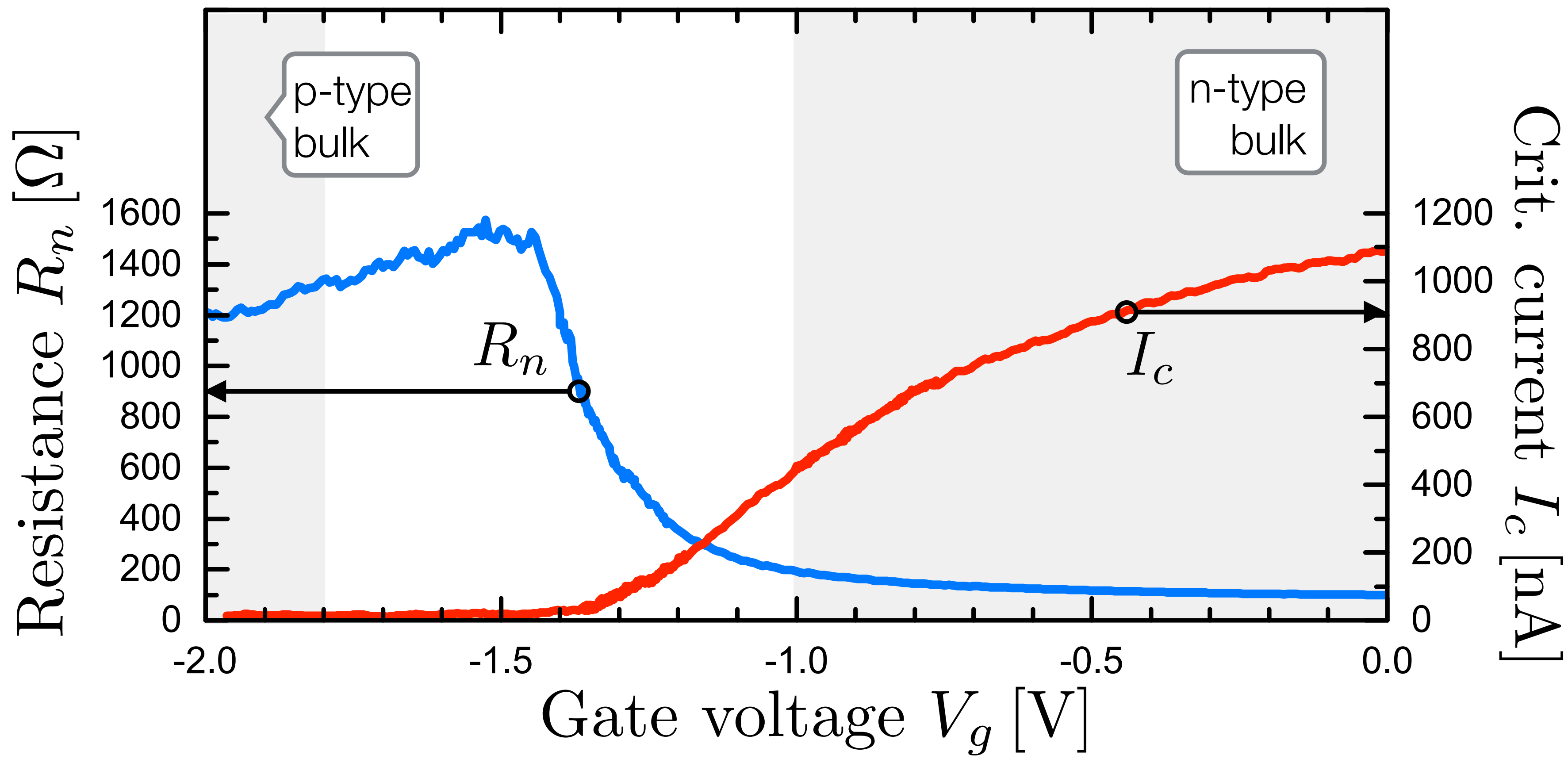
Josephson junctions

- $\mu \simeq 3\ 10^5\ cm^2V^{-1}s^{-1}$
- Al contacts (in situ)
- HfO_2 /Au gate
- no overlap of edge states
- ballistic / intermediate

$$L \ll l \qquad L \lesssim \xi$$

II-B. First properties of HgTe JJs





I-V curve

- weak hysteresis visible
- excess current
  ⇒ Andreev reflections

Gate dependence

- 3 regimes : *p*, *n*, and QSH
- asymmetry between *n* and *p*

Blonder *et al.*, PRB **25**, 4515 (1982)

Interference patterns

Interference patterns

- ▹ non-uniform phase

$$\phi \rightarrow \phi + \frac{2\pi BL}{\Phi_0} y$$

- ▹ « interference » pattern

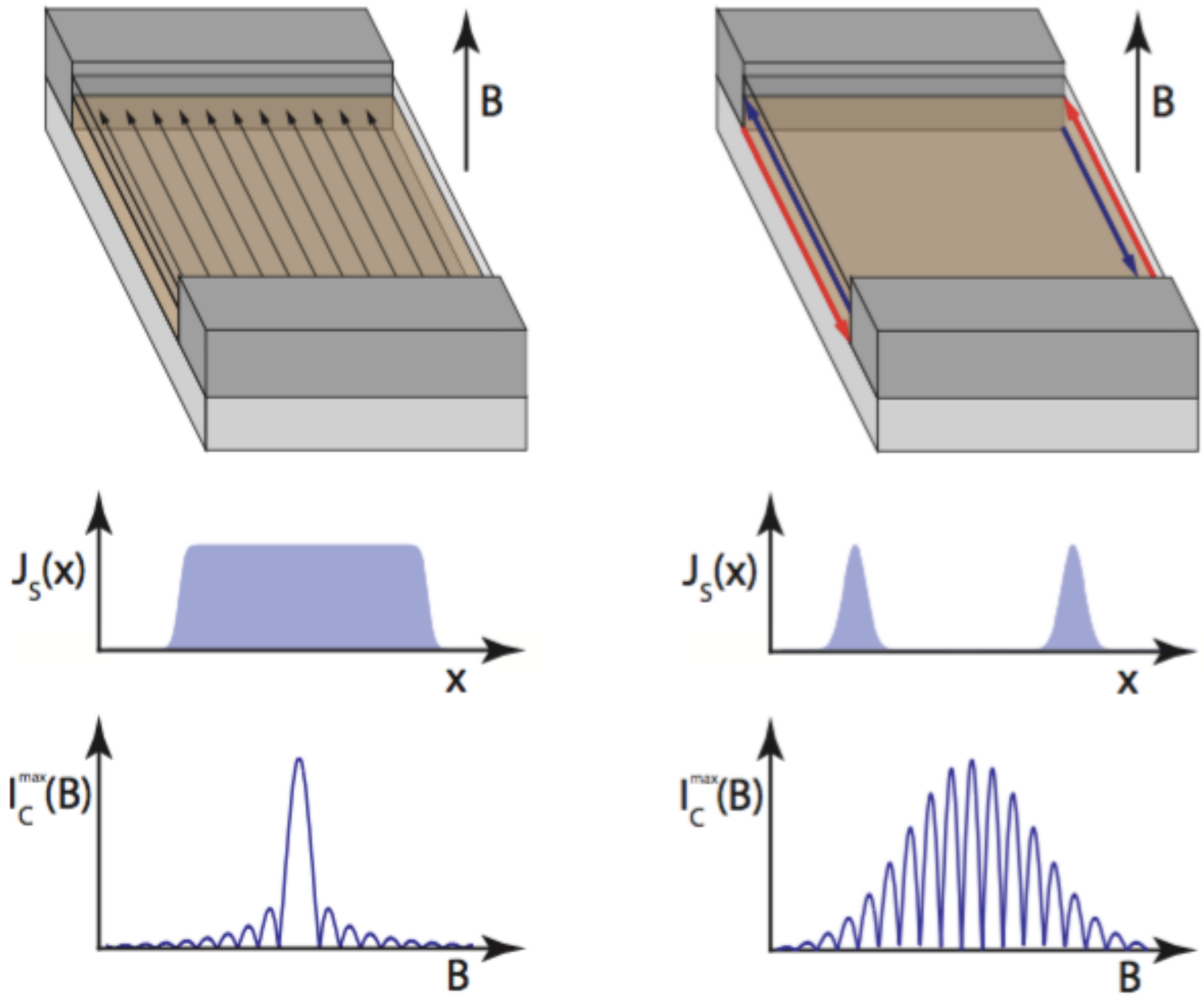
Standard patterns

- ▹ Fraunhofer pattern

  ⇔ uniform supercurrent

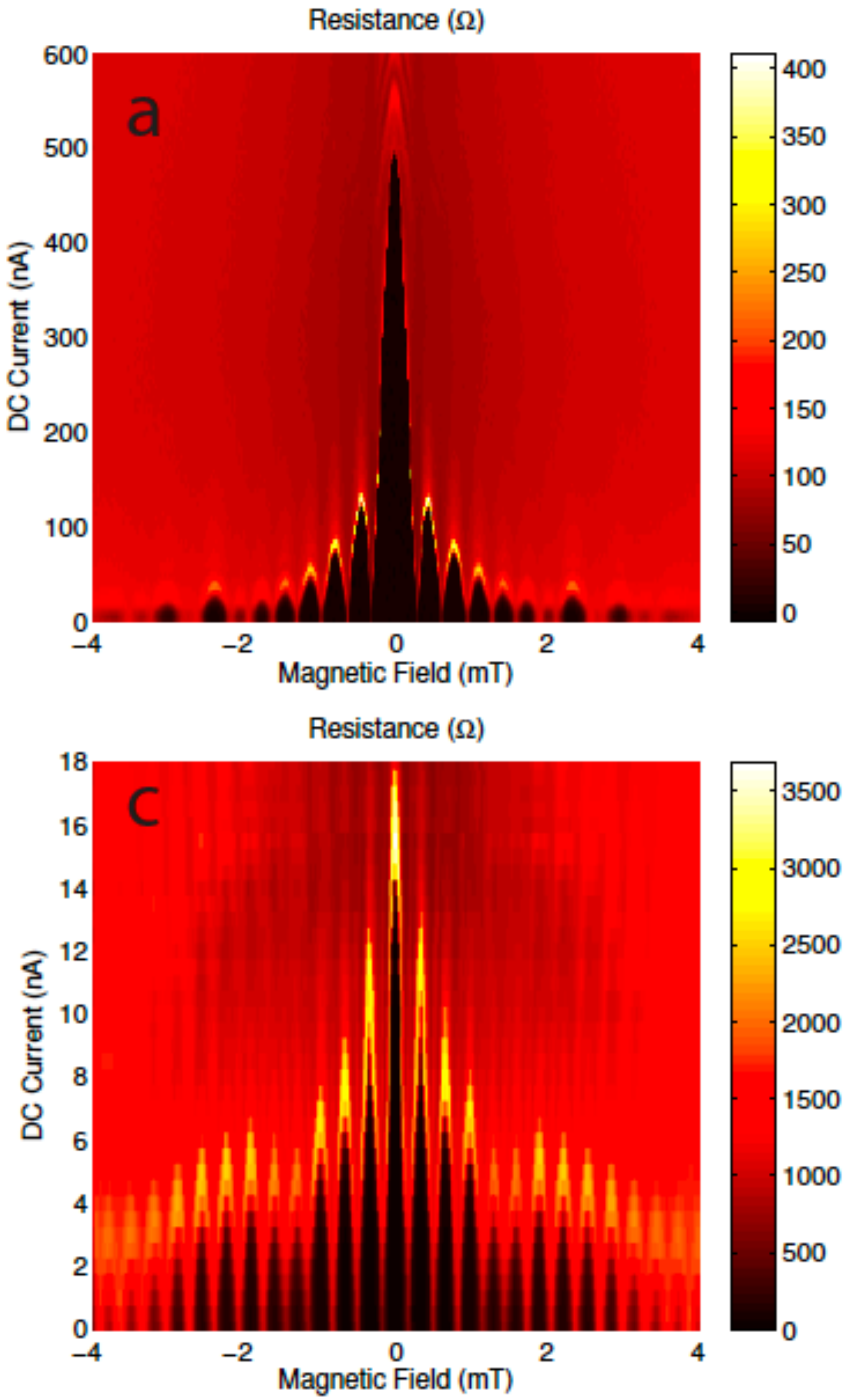
- ▹ SQUID

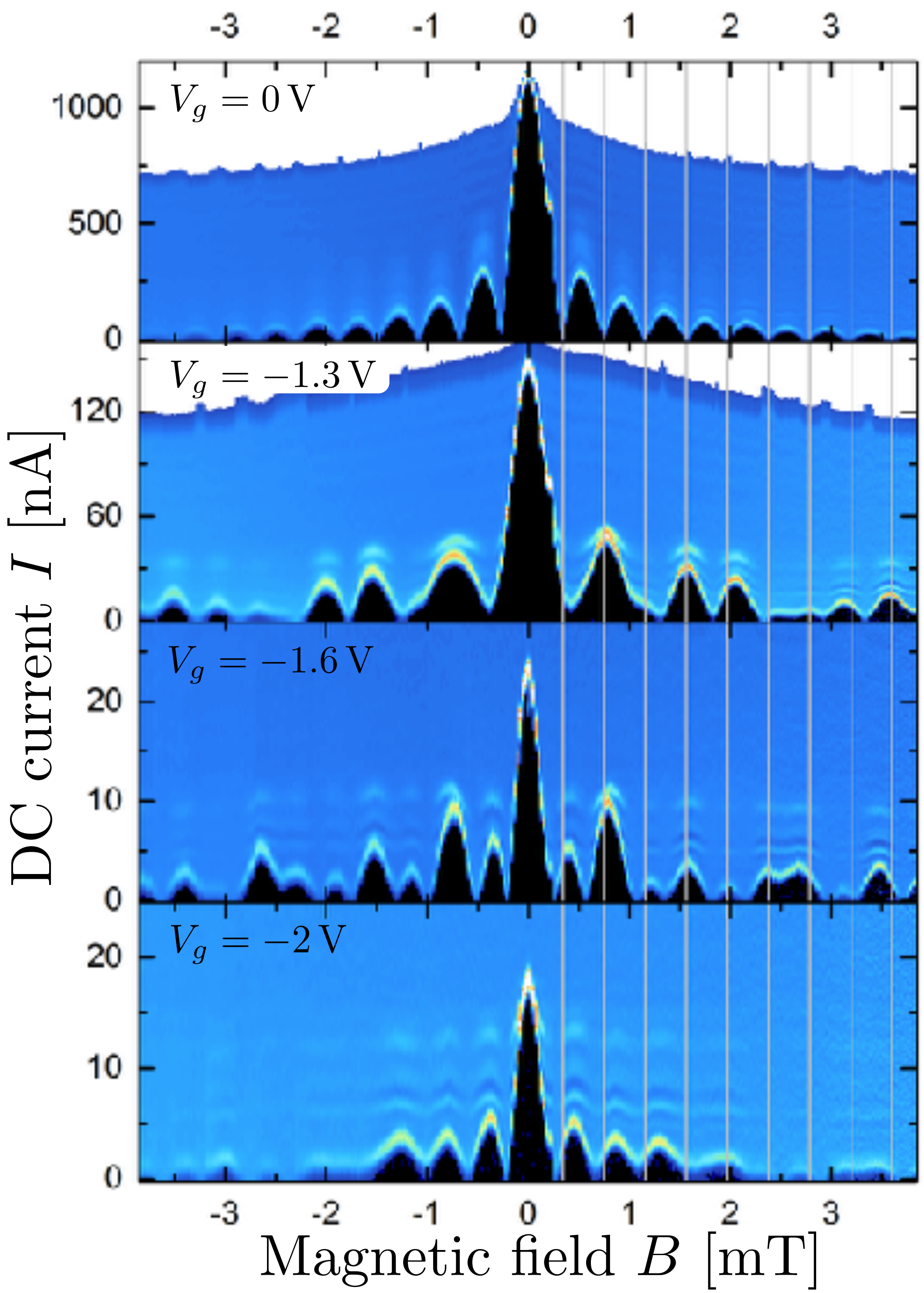
  ⇔ edge currents



Hart *et al.*, Nat. Phys. **10**, 638 (2014)

Anomalous interference patterns

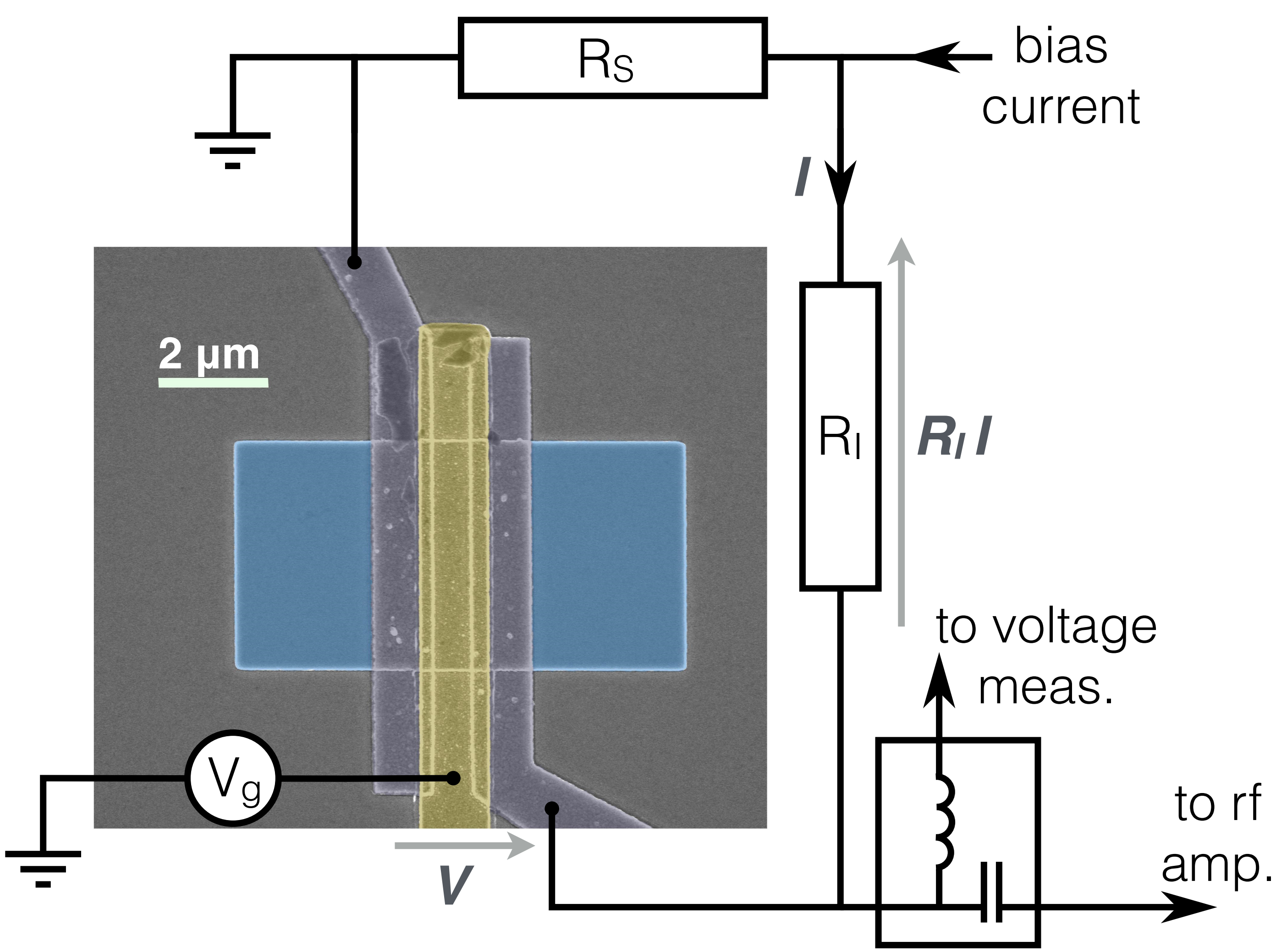


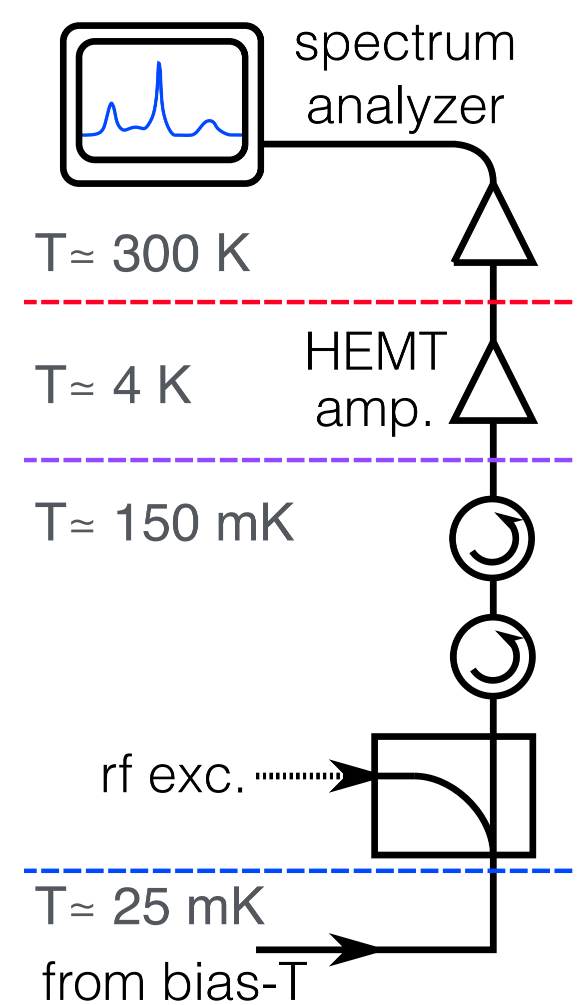


Hart *et al.*, Nat. Phys. **10**, 638 (2014)
Pribiag *et al.*, Nat. Nano. **10** 593 (2015)

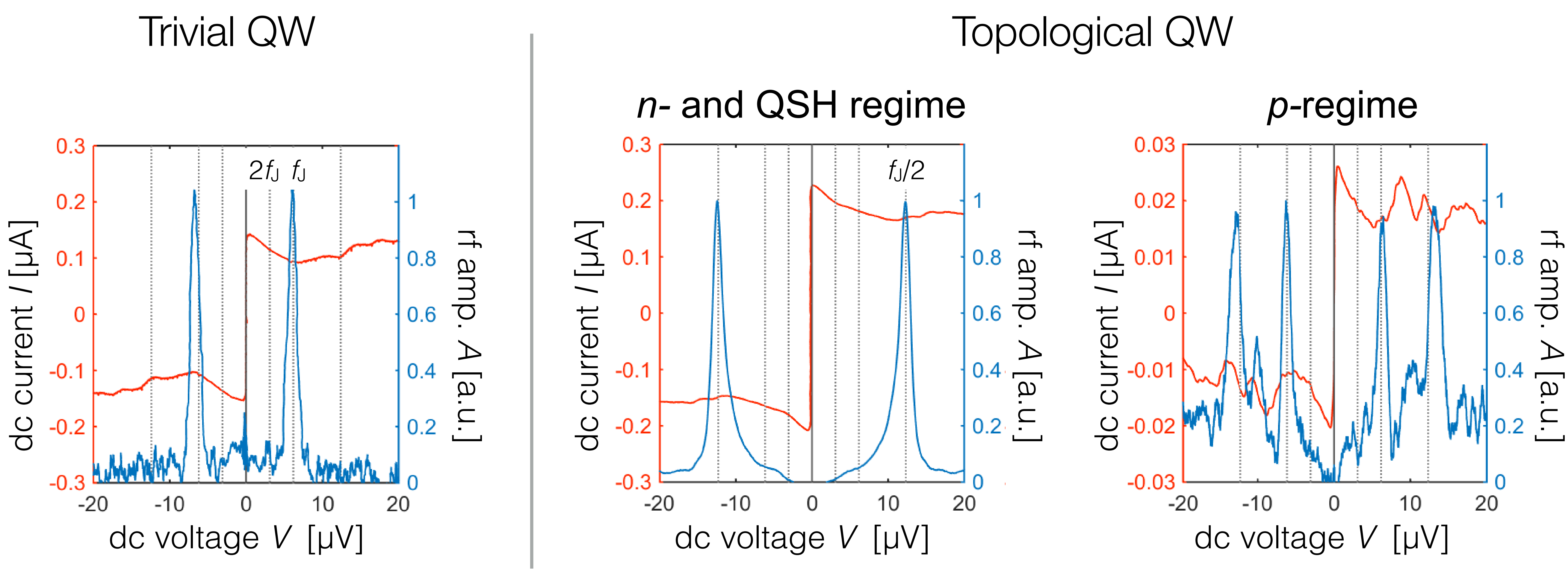
Detection setup

- voltage bias
  - shunt resistance R_S
  - current resistance R
- rf amplification setup
  - 1 cryo amp. (+ 2 amps at RT)
  - 0.1 fW (-130 dBm) in 8 MHz





Emission spectra

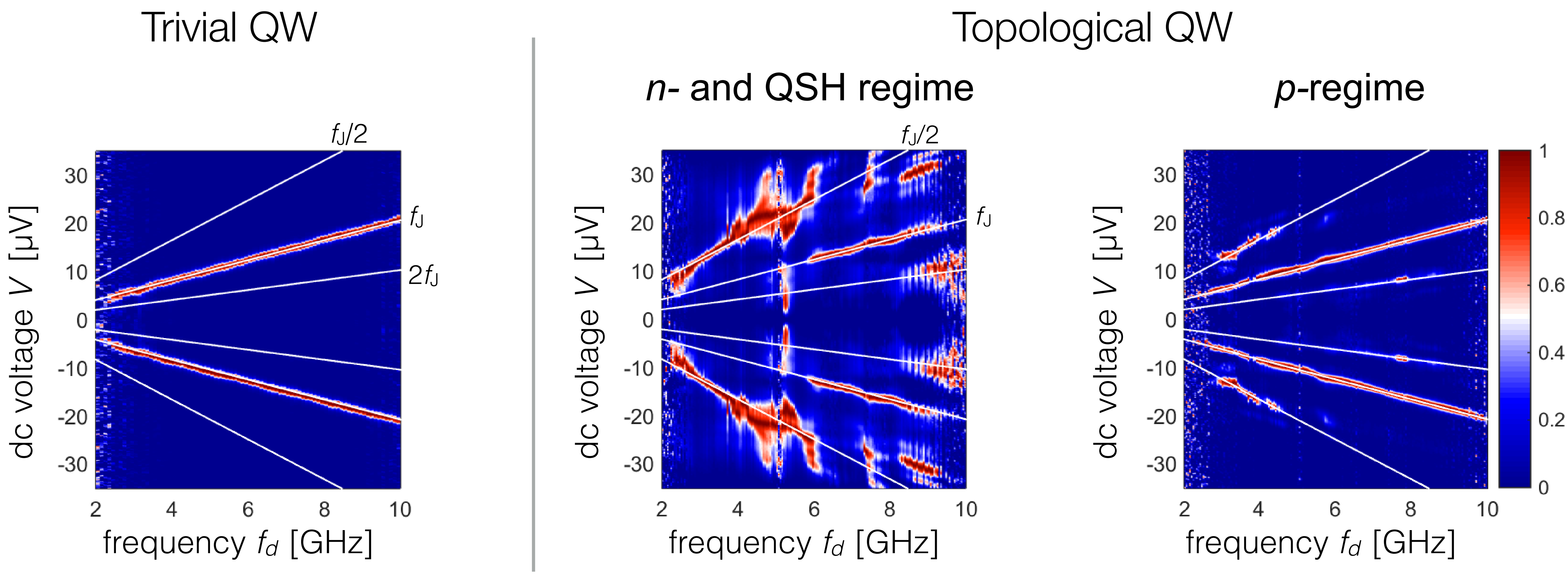


- voltage V swept
- integrated power at f_d=3 GHz (in 8 MHz bandwidth)

- trivial QW : signal at $f_d=f_J$
- topological QW : at $f_d=f_J$ and $f_J/2$

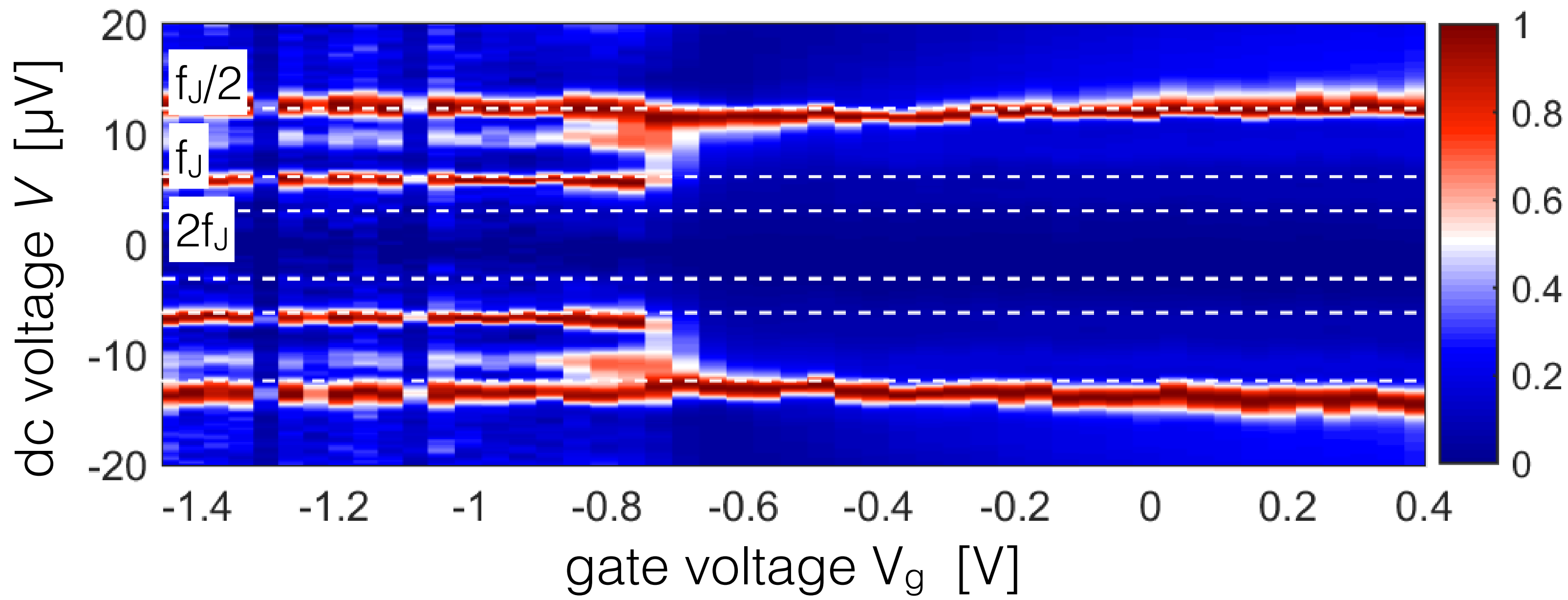
Deacon *et al.*, submitted, ArXiv 1603.09611 (2016)

Frequency dependence



- $f_J = 2eV/h$
- Stronger $f_J/2$ signal at low frequencies
- Relative intensities of $f_J/2$ and f_J depending on V_g

Gate voltage dependence



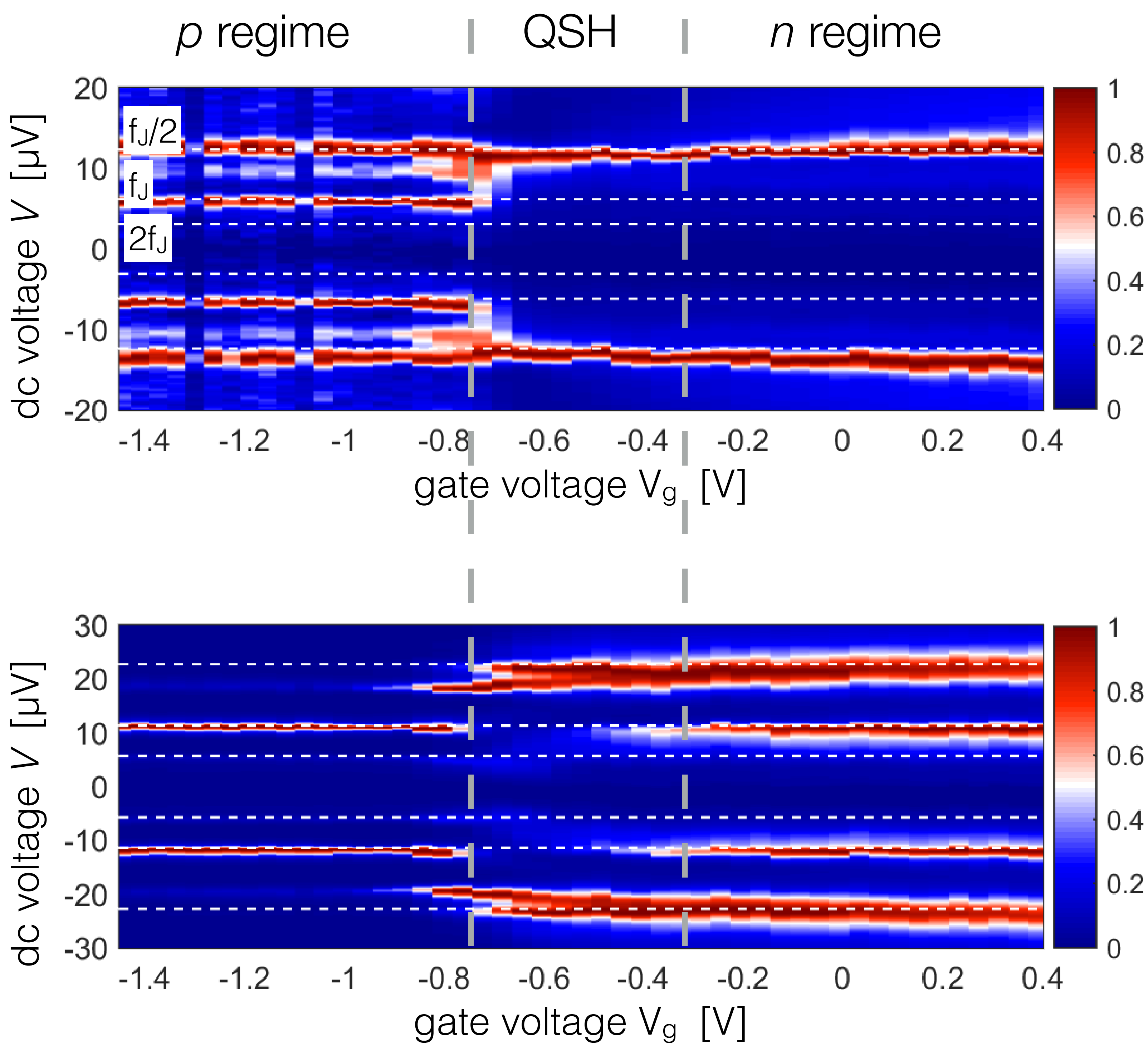
Low frequency

f_d= 3 GHz

High frequency

f_d= 5.5 GHz

Gate voltage dependence



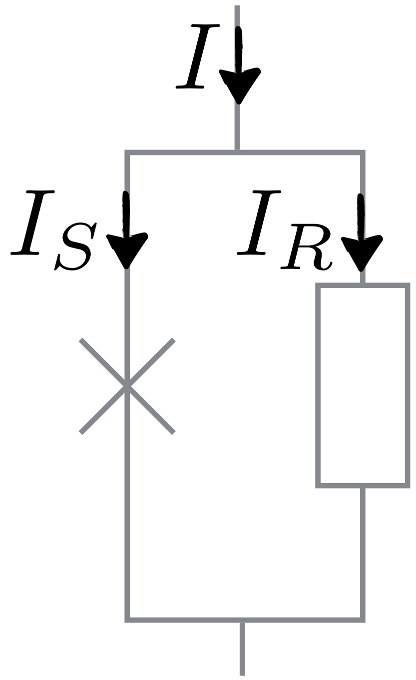
Low frequency

f_d= 3 GHz

High frequency

f_d= 5.5 GHz

RSJ model : DC current bias



Josephson/RSJ equations

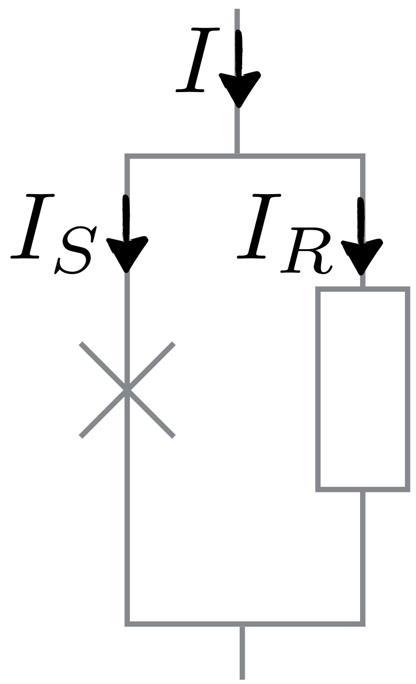
- $\dot{\phi} = \frac{2eV}{\hbar}$

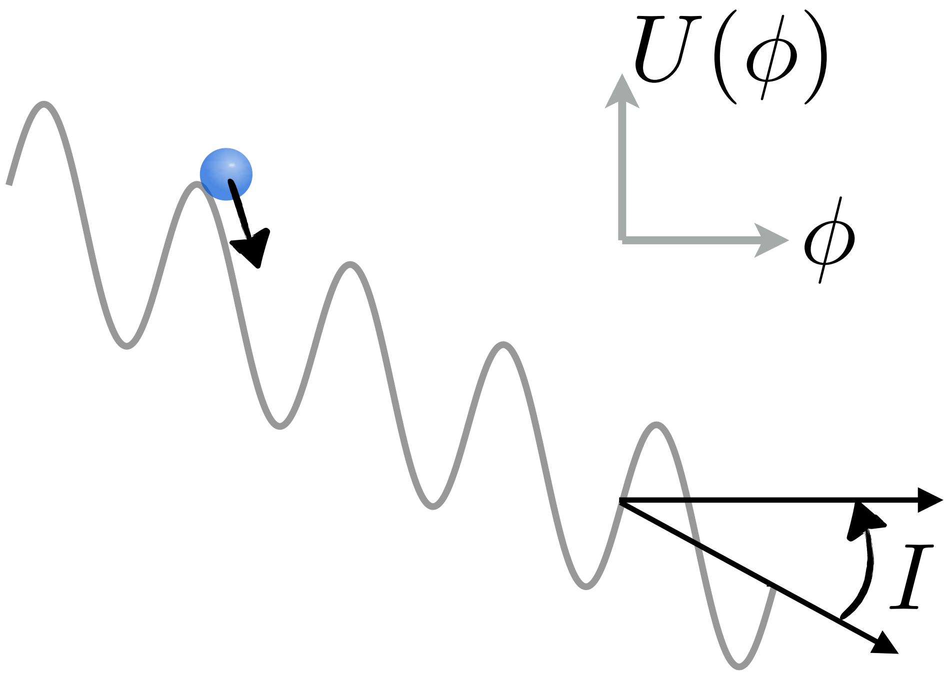
  $I = I_S(\phi) + \frac{\hbar}{2eR}\dot{\phi}$

$$I_S(\phi) = I_{4\pi} \sin\frac{\phi}{2} + I_{2\pi} \sin\phi + \ldots$$

Stewart, APL **12**, 277 (1968)
McCumber, J. App. Phys. **39**, 3113 (1968)

RSJ model : DC current bias





Josephson/RSJ equations

- $\dot{\phi} = \frac{2eV}{\hbar}$

  $I = I_S(\phi) + \frac{\hbar}{2eR}\dot{\phi}$

$$I_S(\phi) = I_{4\pi} \sin\frac{\phi}{2} + I_{2\pi}\sin\phi + \dots$$

Stewart, APL **12**, 277 (1968)
McCumber, J. App. Phys. **39**, 3113 (1968)

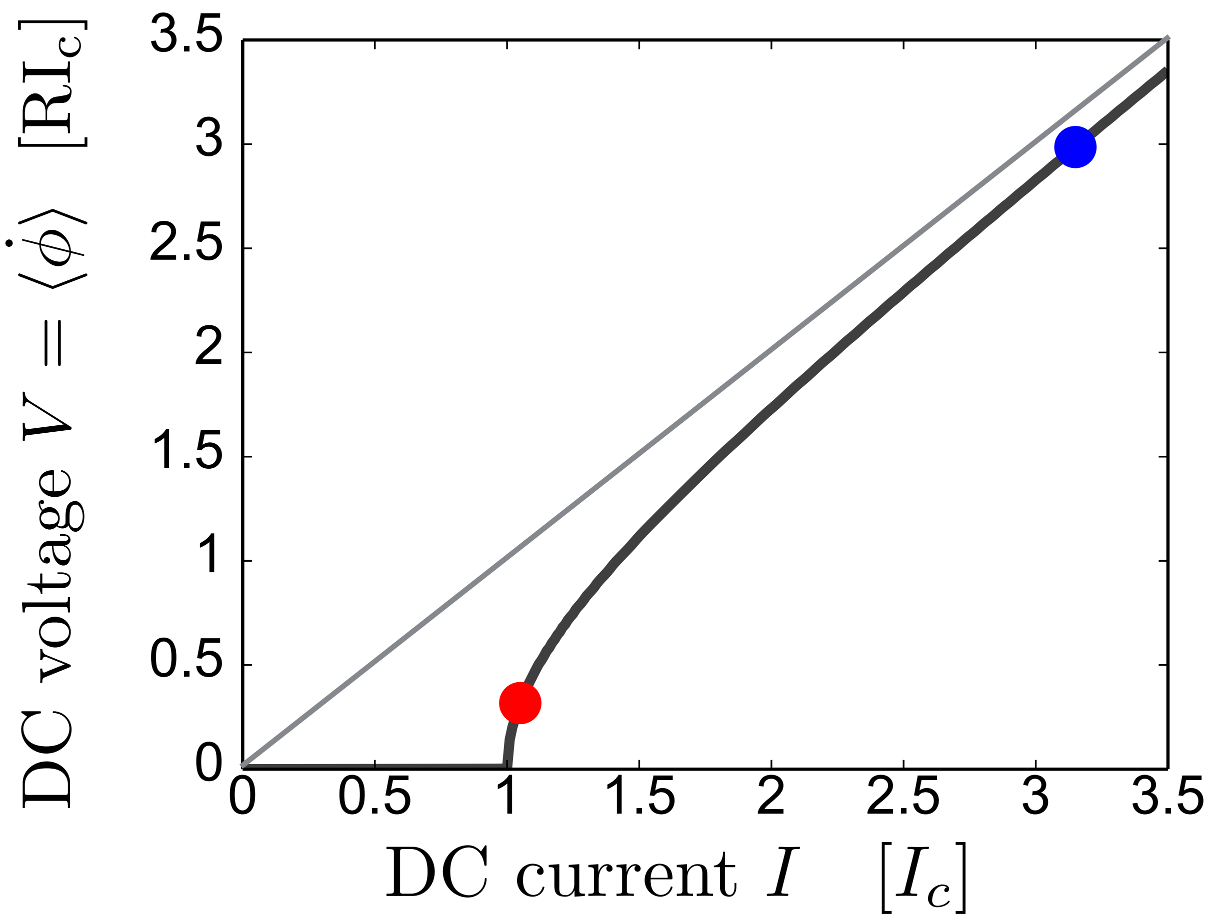
Motion of fictitious particle

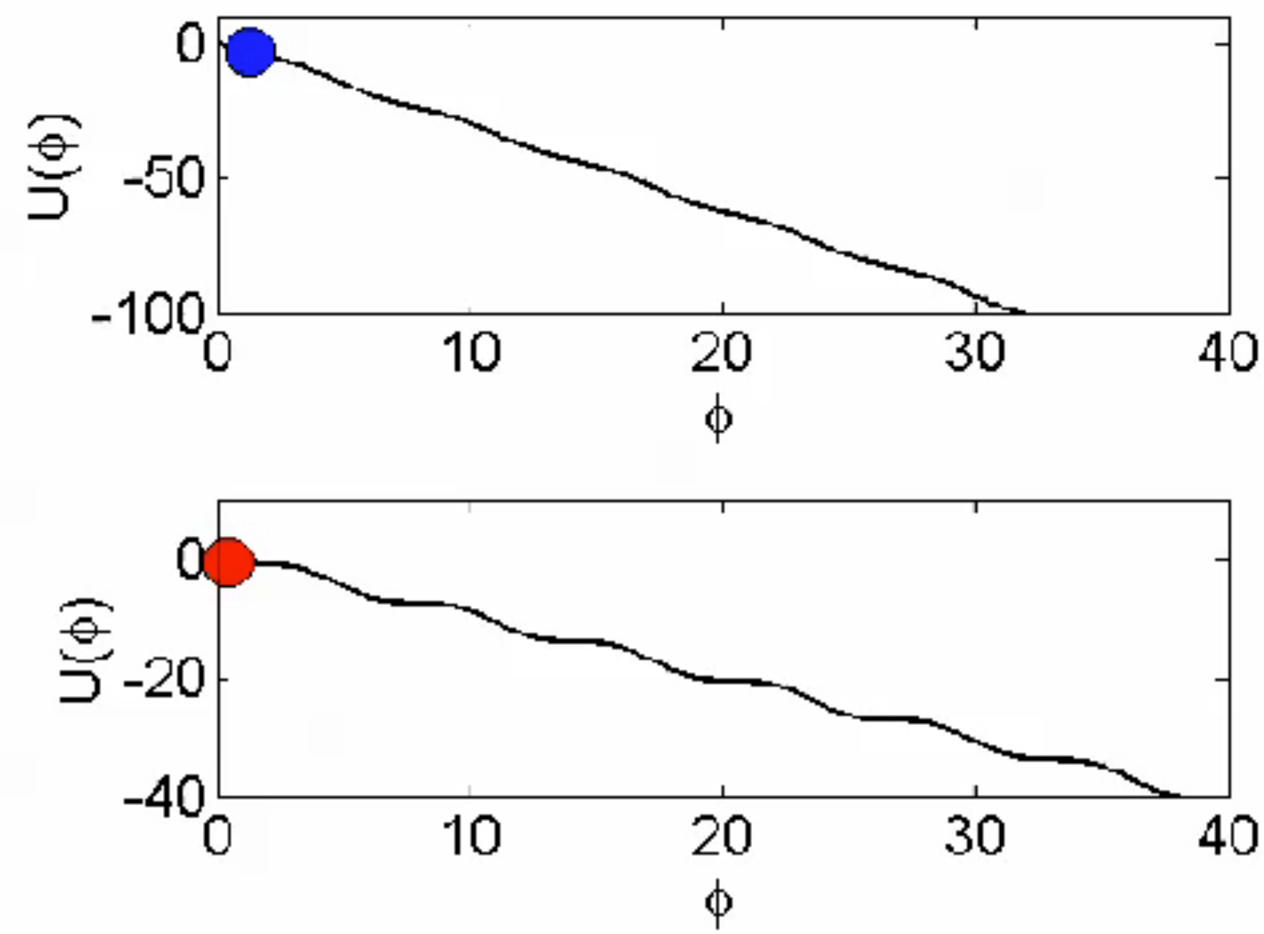
- $\partial_\phi U(\phi) = I_S(\phi) - I$
- Zero-voltage state $I < I_c$

  $\phi = \mathrm{C}^{\mathrm{st}} \longleftrightarrow$ particle trapped
- Voltage state $I > I_c$

  $\phi \nearrow \longleftrightarrow$ particle falling down

I-V curve from RSJ model

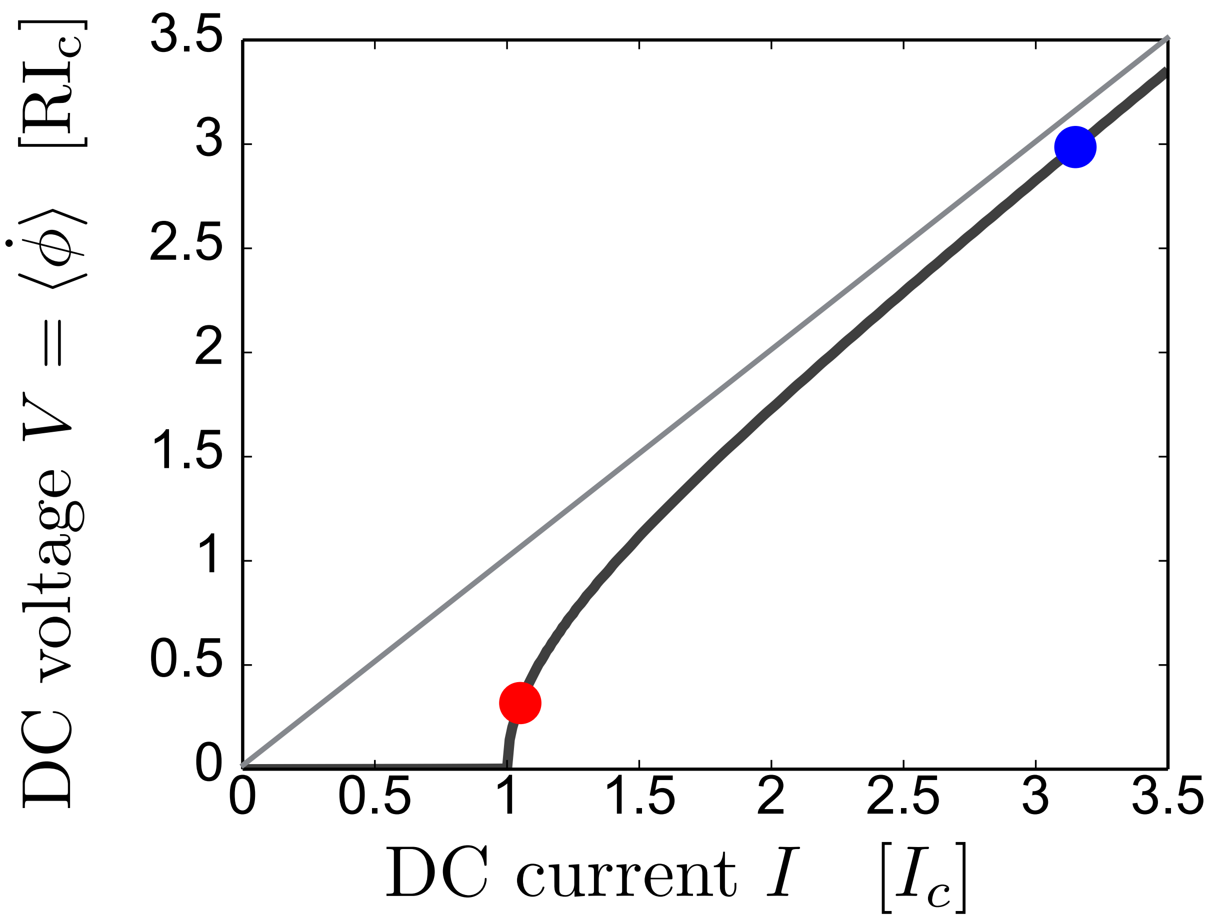


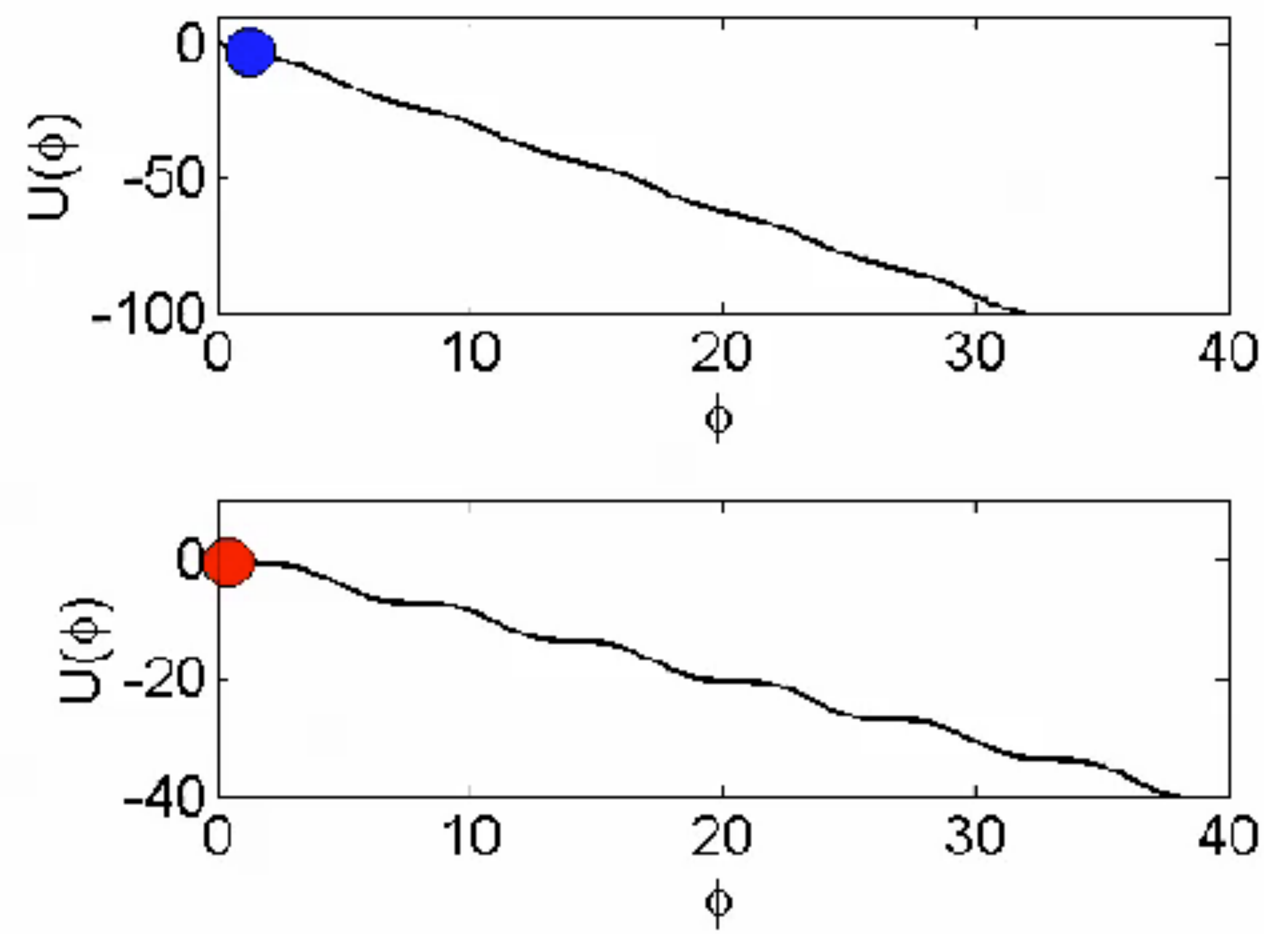


$$\overline{V} = RI\sqrt{1-(I_c/I)^2}$$

- Harmonic for $I \gg I_c$
  Anharmonic for $I \simeq I_c$
- Frequency $f_\mathrm{J} = \dfrac{2e\overline{V}}{h}$

I-V curve from RSJ model





$$\overline{V} = RI\sqrt{1-(I_c/I)^2}$$

- Harmonic for $I \gg I_c$
  Anharmonic for $I \simeq I_c$
- Frequency $f_\mathrm{J} = \dfrac{2e\overline{V}}{h}$

AC response : Shapiro steps

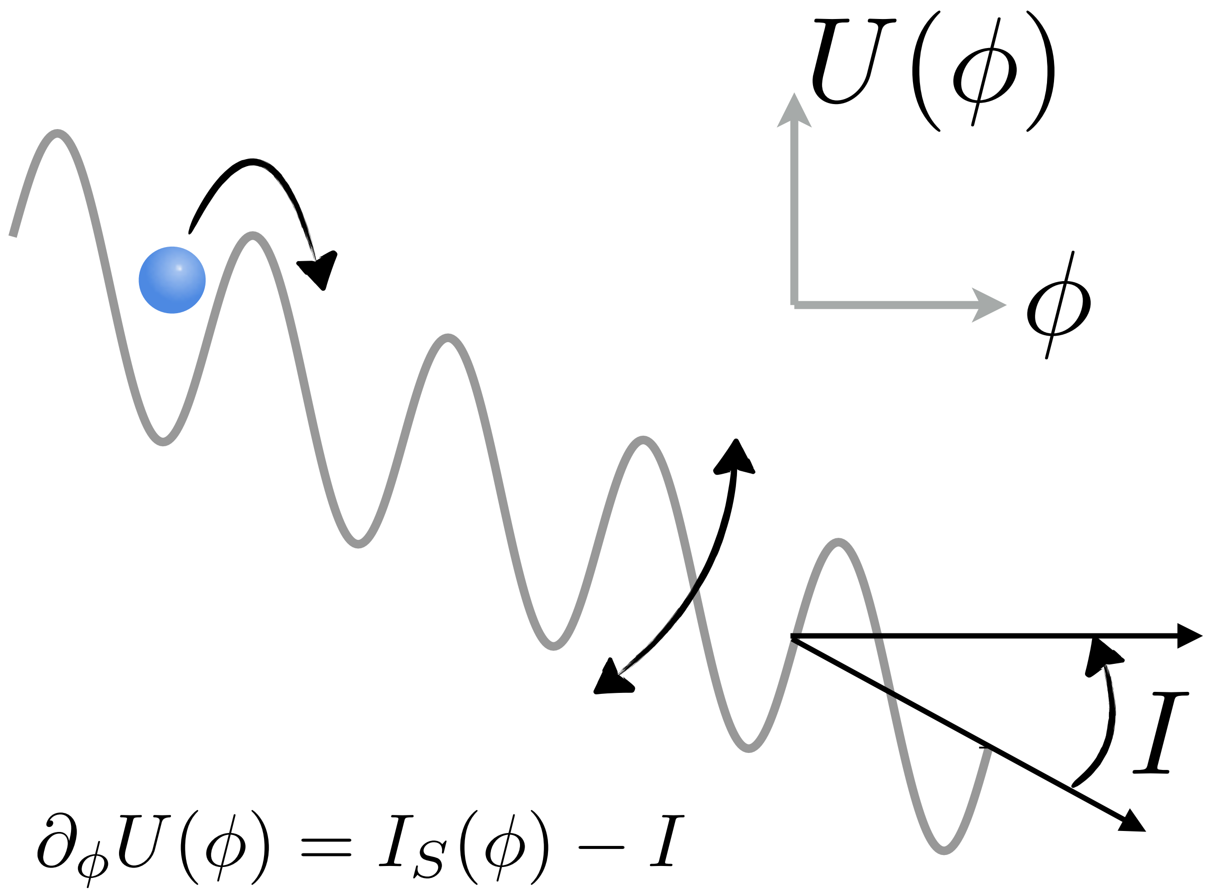
Phase-locked motion

- phase dynamics (RSJ model)

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$
$$I = I_S(\phi) + \frac{\hbar}{2eR}\dot{\phi}$$

- motion locked to rf excitation

$$\frac{\Delta\phi}{\Delta t} = \frac{2\pi n}{1/f} \Rightarrow V_n = n\frac{hf}{2e}$$



Shapiro, PRL **11**, 80 (1963)
Russer, J. App. Phys. **43**, 2008 (1972)

AC response : Shapiro steps

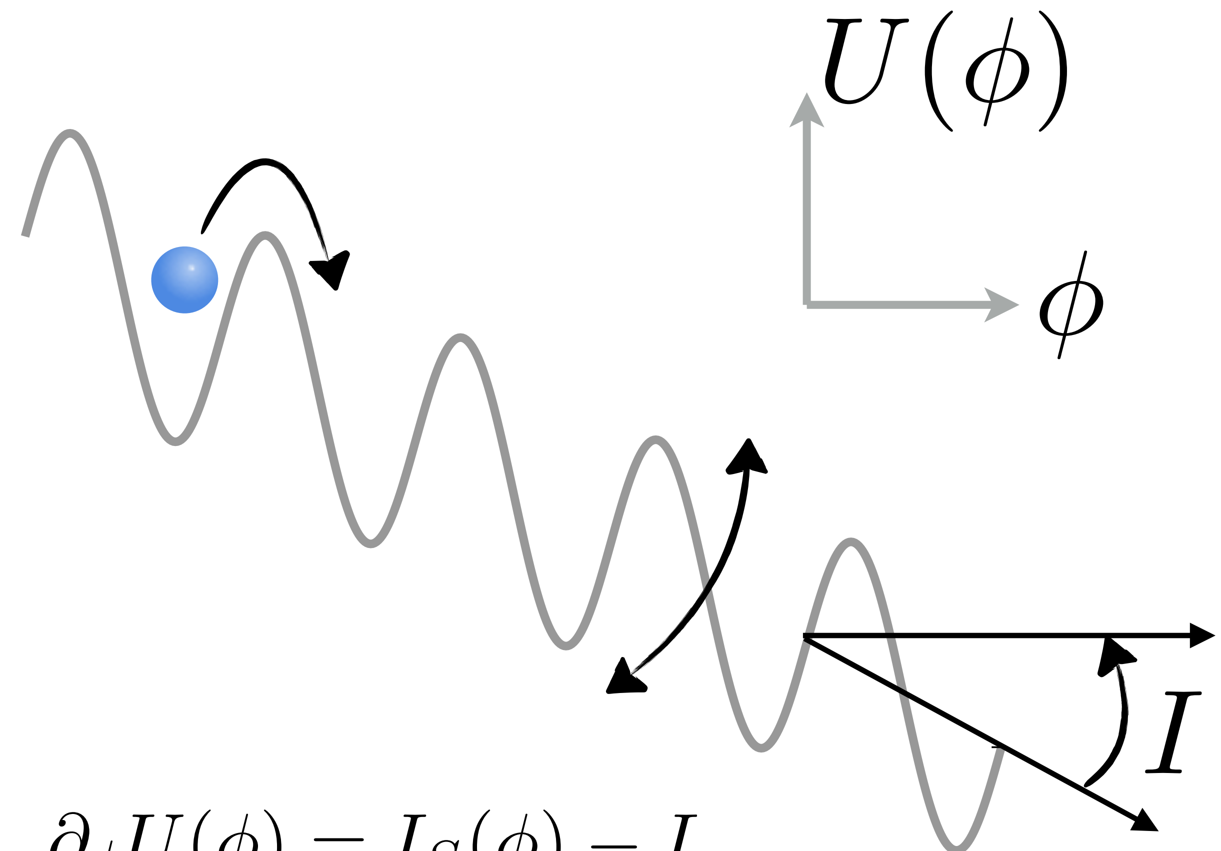
Phase-locked motion

- ▹ phase dynamics (RSJ model)

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$
$$I = I_S(\phi) + \frac{\hbar}{2eR}\dot{\phi}$$

- ▹ motion locked to rf excitation

$$\frac{\Delta\phi}{\Delta t} = \frac{2\pi n}{1/f} \Rightarrow V_n = n\frac{hf}{2e}$$



$$\partial_\phi U(\phi) = I_S(\phi) - I$$

Shapiro, PRL **11**, 80 (1963)
Russer, J. App. Phys. **43**, 2008 (1972)

AC response : Shapiro steps

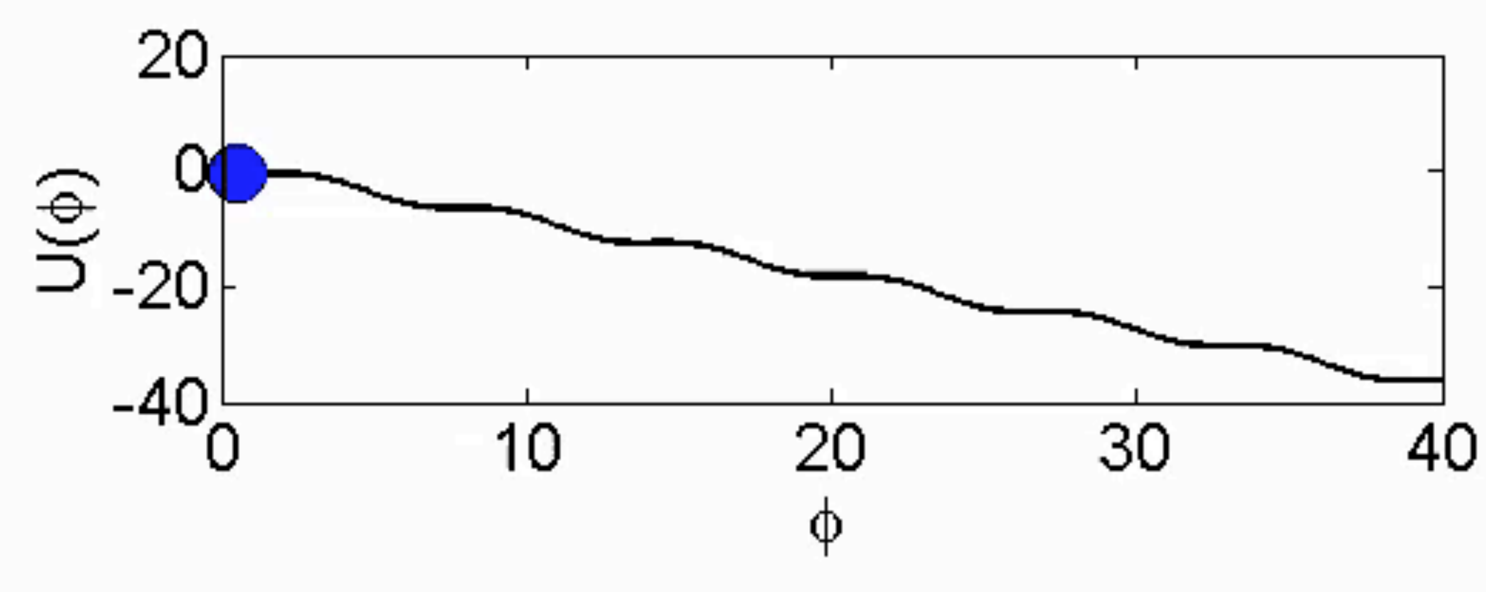
Phase-locked motion

- phase dynamics (RSJ model)

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$
$$I = I_S(\phi) + \frac{\hbar}{2eR}\dot{\phi}$$

- motion locked to rf excitation

$$\frac{\Delta\phi}{\Delta t} = \frac{2\pi n}{1/f} \Rightarrow V_n = n\frac{hf}{2e}$$



Shapiro, PRL **11**, 80 (1963)
Russer, J. App. Phys. **43**, 2008 (1972)

AC response : Shapiro steps

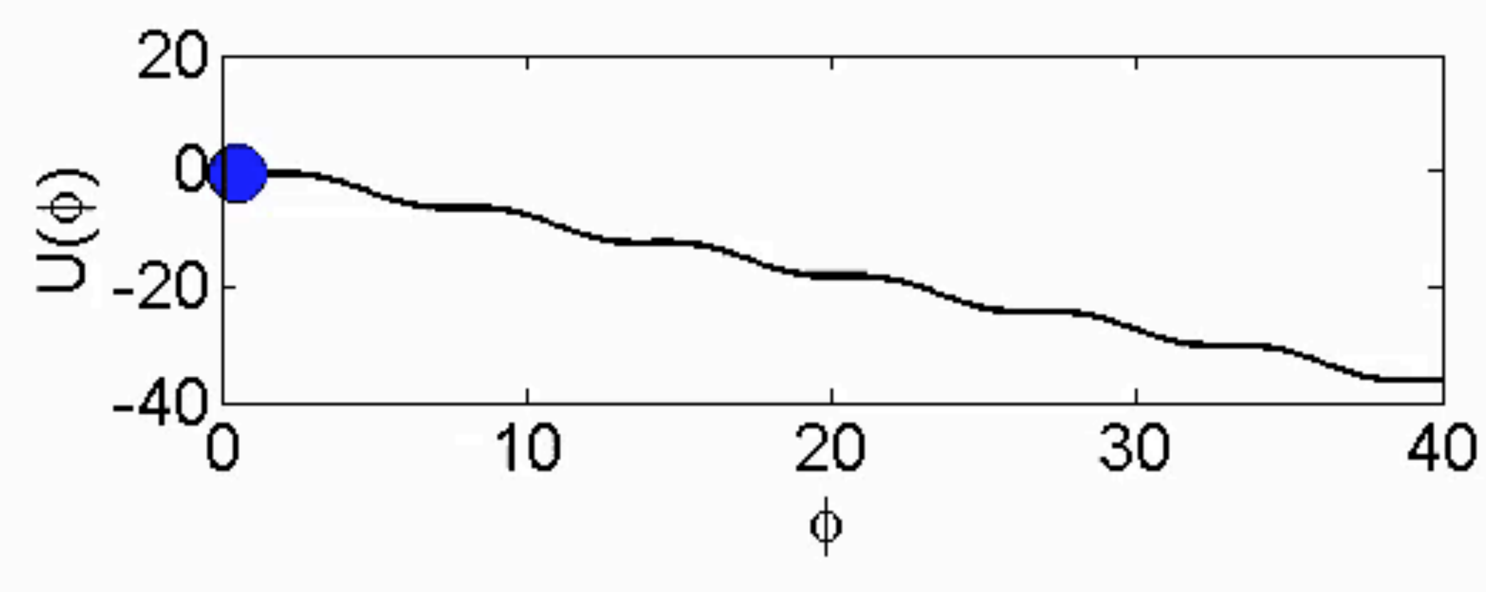
Phase-locked motion

- phase dynamics (RSJ model)

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$
$$I = I_S(\phi) + \frac{\hbar}{2eR}\dot{\phi}$$

- motion locked to rf excitation

$$\frac{\Delta\phi}{\Delta t} = \frac{2\pi n}{1/f} \Rightarrow V_n = n\frac{hf}{2e}$$



4π-periodic supercurrent

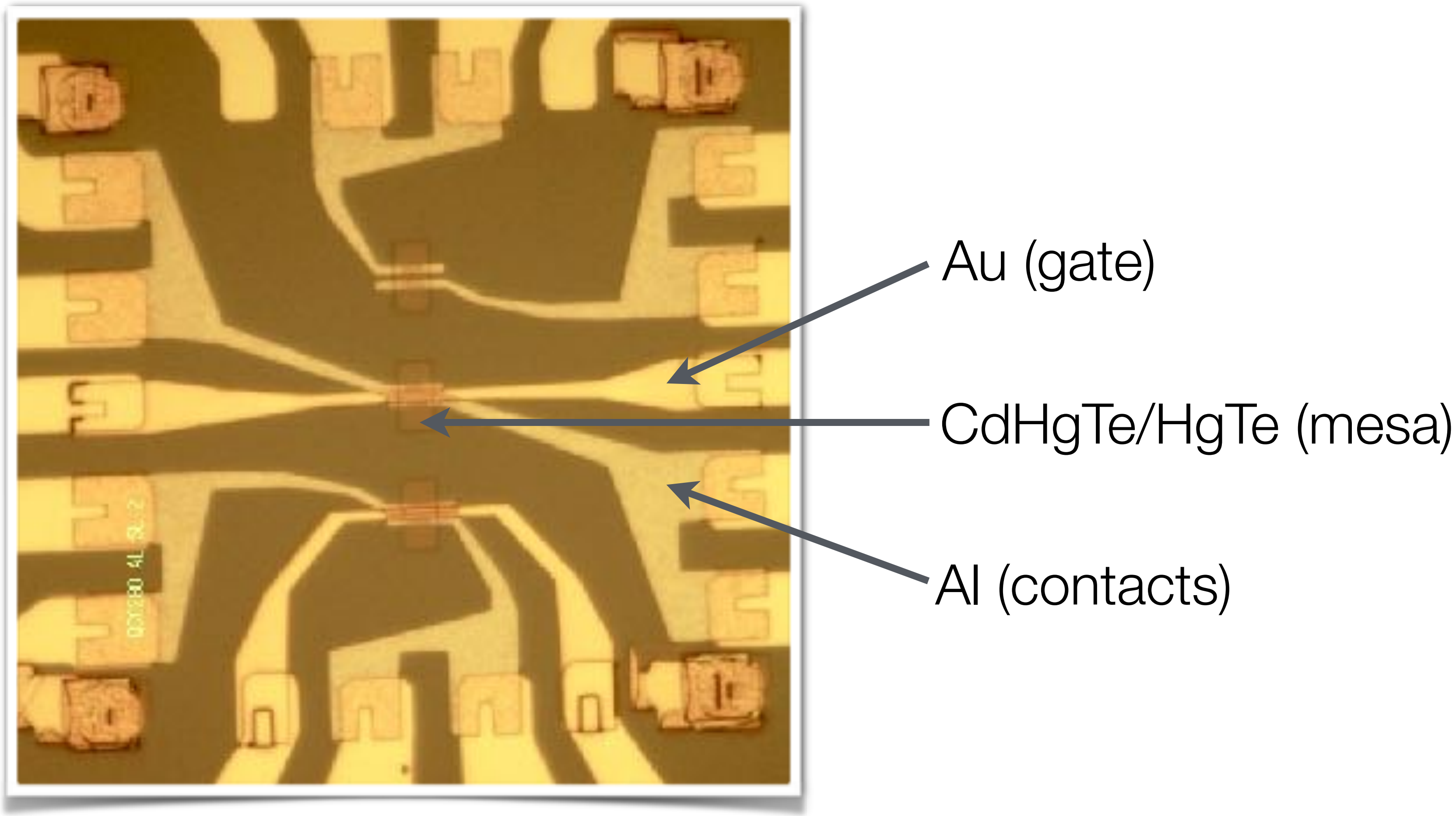
- doubled steps

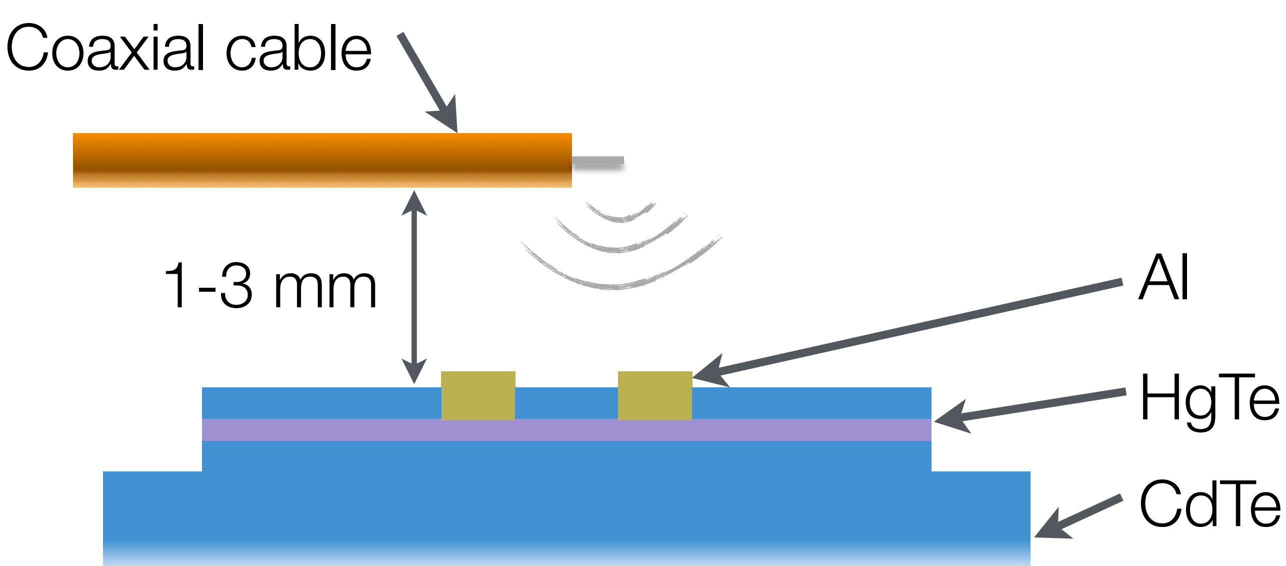
$$\sin\phi \to \sin\phi/2$$
$$V_n \to V_{2n}$$

- mixture 2π/4π ?

Shapiro, PRL **11**, 80 (1963)
Russer, J. App. Phys. **43**, 2008 (1972)

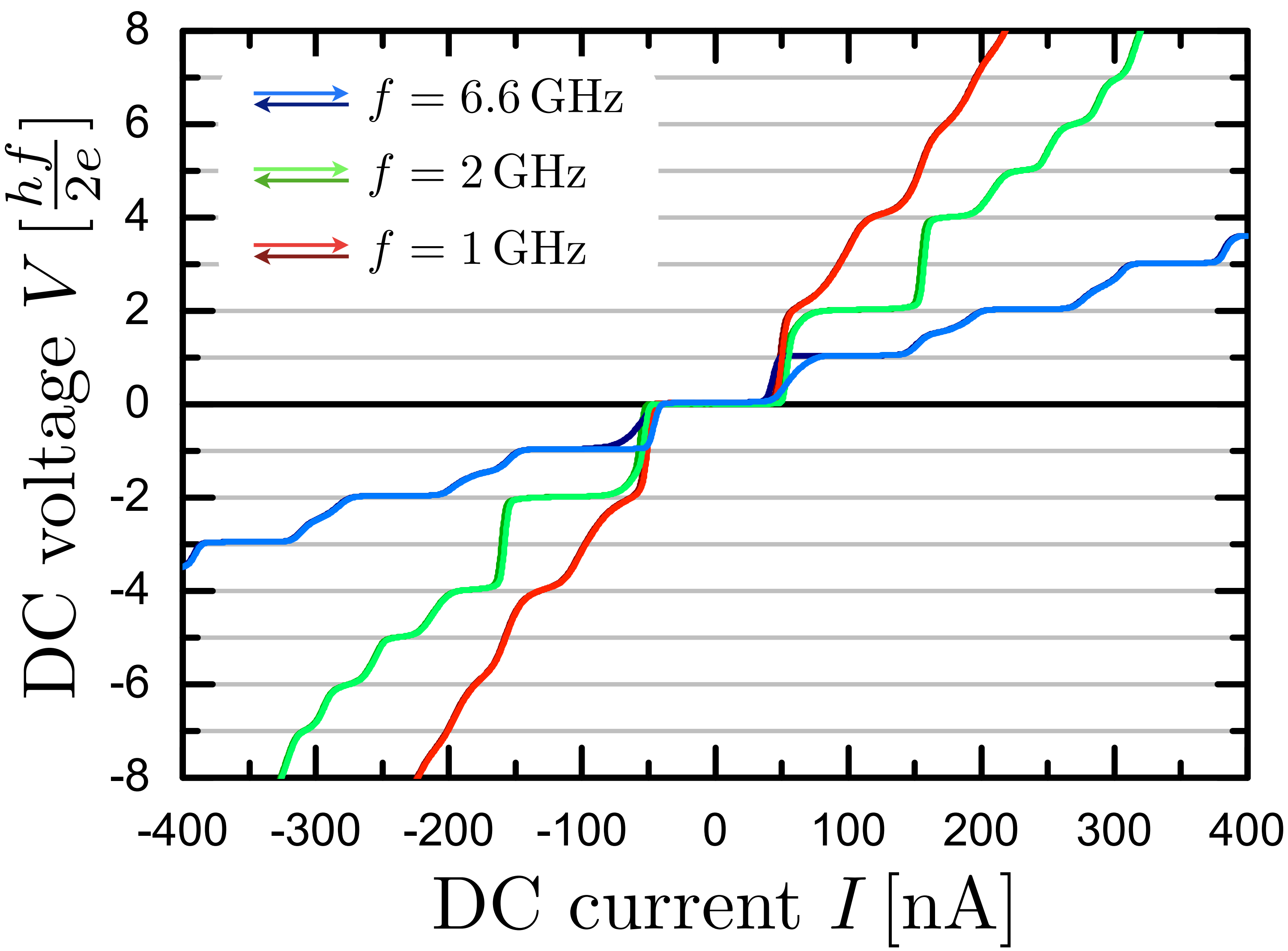
Experimental realization





- open-ended coaxial cable
- frequency range $f \sim 0.5 - 12\,\mathrm{GHz}$
- 4 devices tested
  2 substrates

Shapiro response : frequency

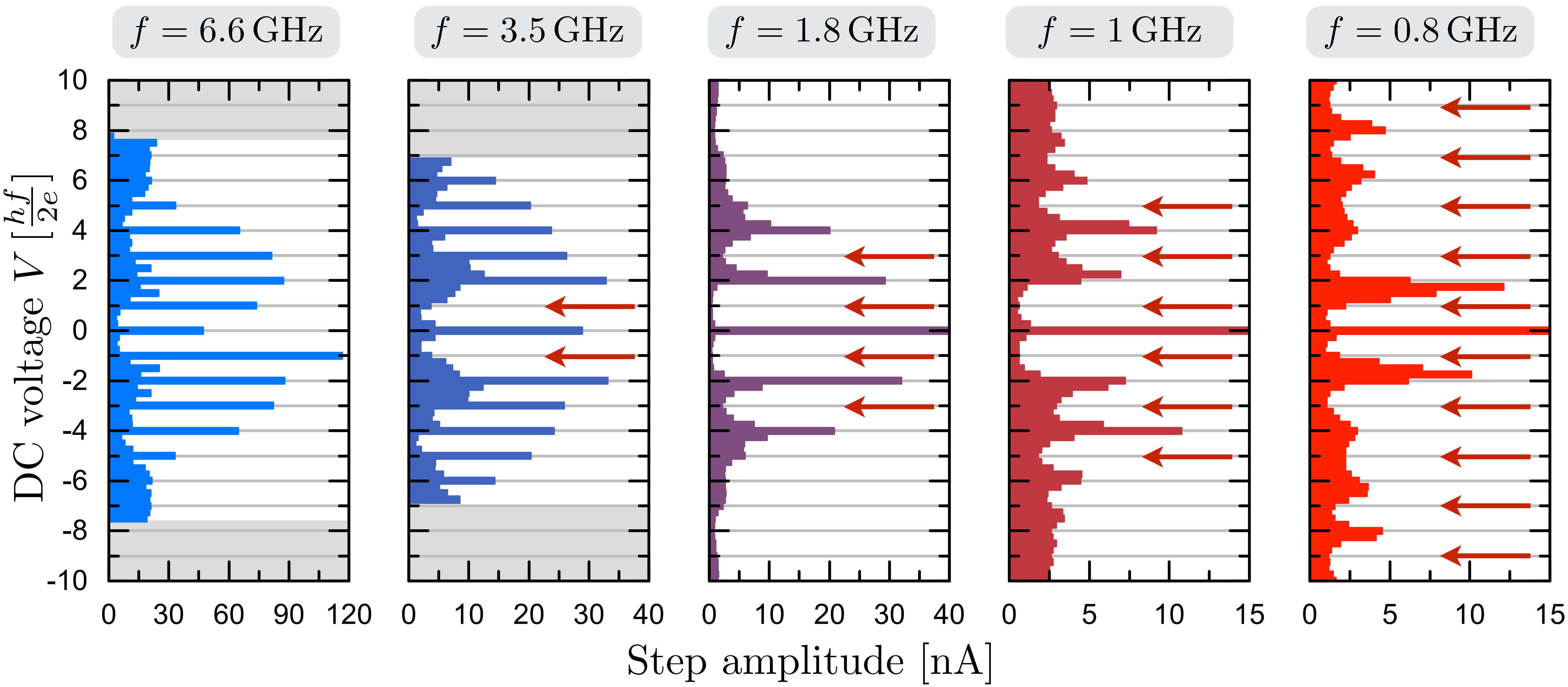


Shapiro response

- >12 steps visible
- weak hysteresis on 1st step
- multiple odd steps missing
- at low frequency f ≲ 4 GHz

Rokhinson *et al.*, Nat. Phys. **86**, 146503 (2012)
Wiedenmann *et al.*, Nat. Comms **7**, 10303 (2016)
Bocquillon *et al.*, Nat. Nano, DOI: 10.1038/NNANO.2016.159

Shapiro response : frequency

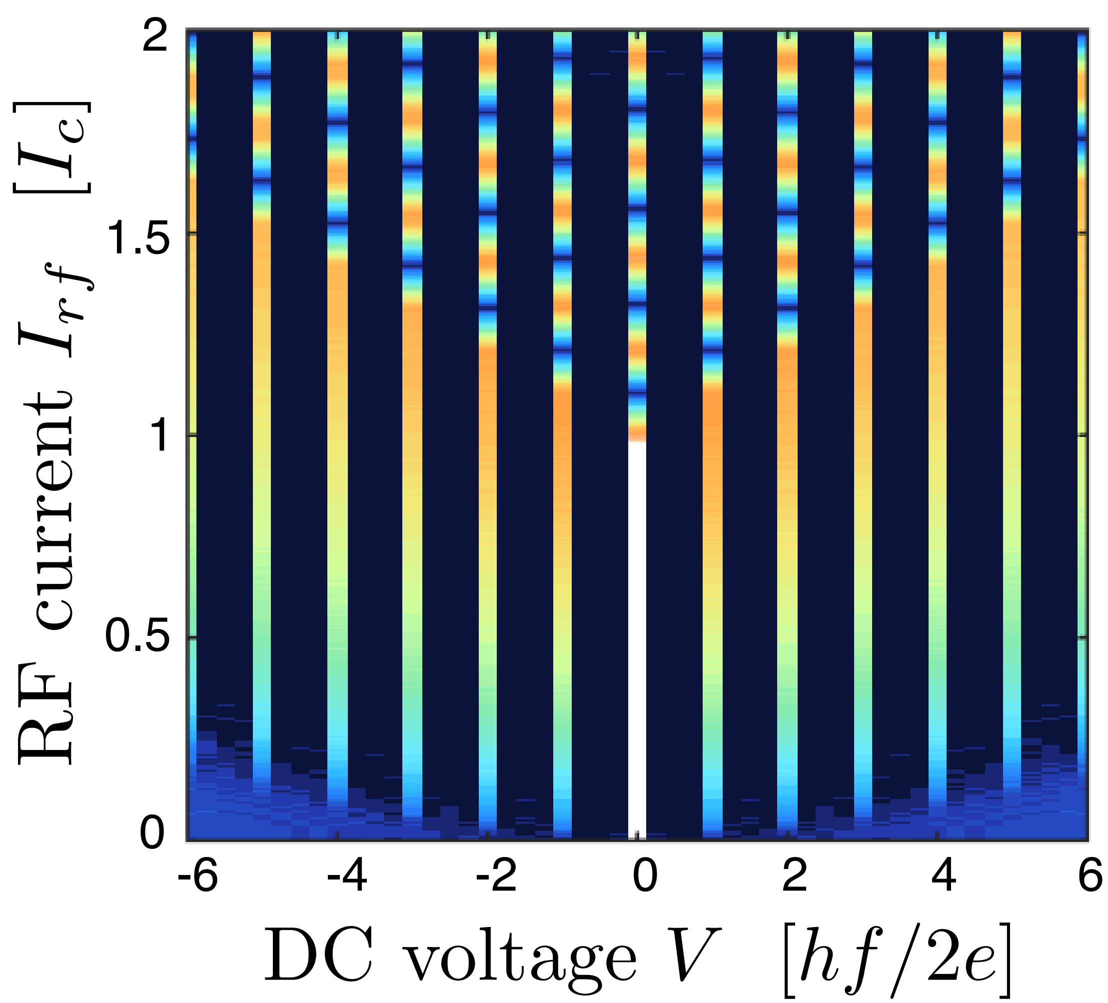


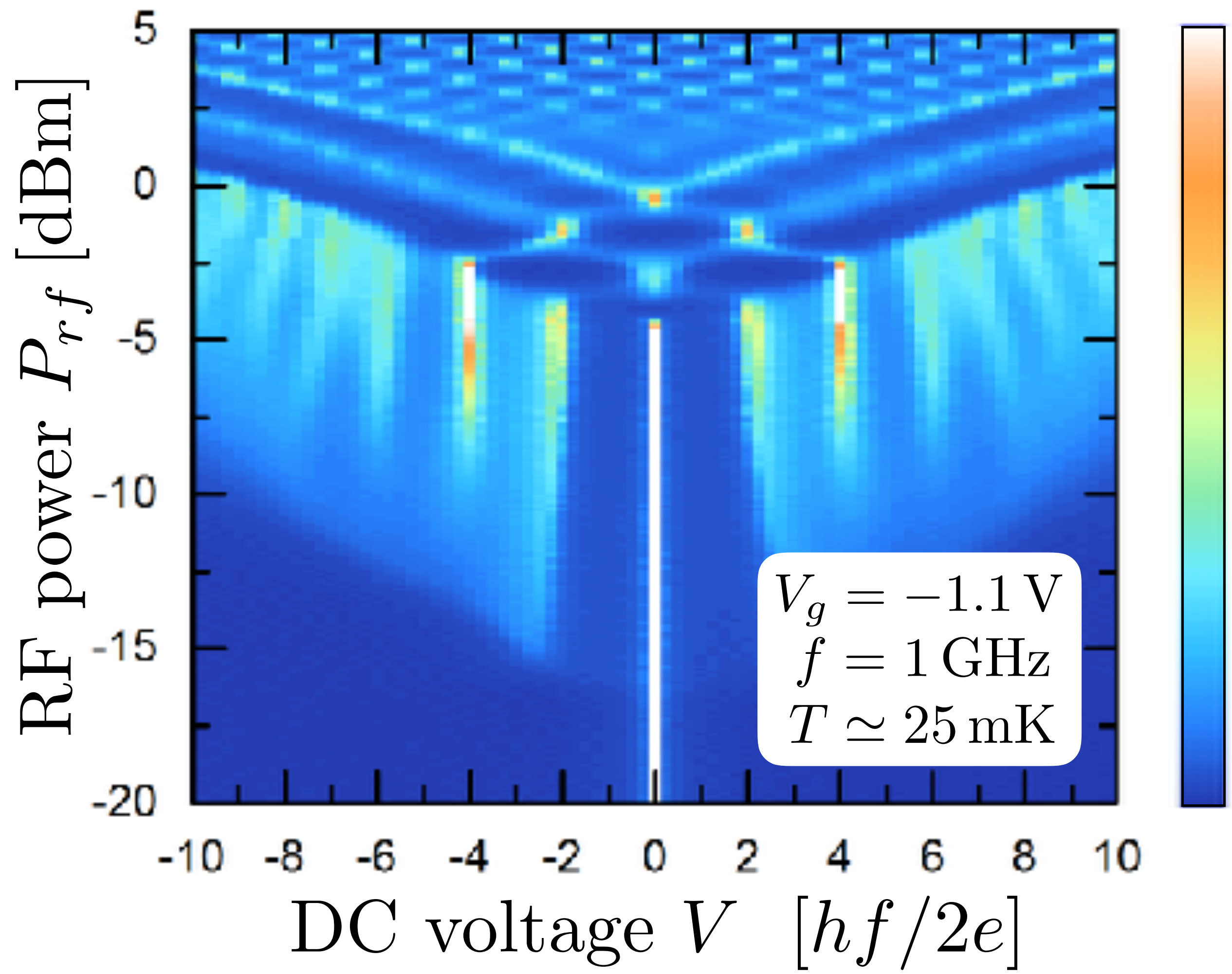
Shapiro response

- ▹ >12 steps visible
- ▹ weak hysteresis on 1st step
- ▹ multiple odd steps missing
- ▹ at low frequency f ≲ 4 GHz

Rokhinson *et al.*, Nat. Phys. **86**, 146503 (2012)
Wiedenmann *et al.*, Nat. Comms **7**, 10303 (2016)
Bocquillon *et al.*, Nat. Nano, DOI: 10.1038/NNANO.2016.159

Shapiro response : power





Simulated response

- at low power : steps forming
- at high power : oscillatory pattern

Our device

- missing n=1,3,5
- « dark fringes »

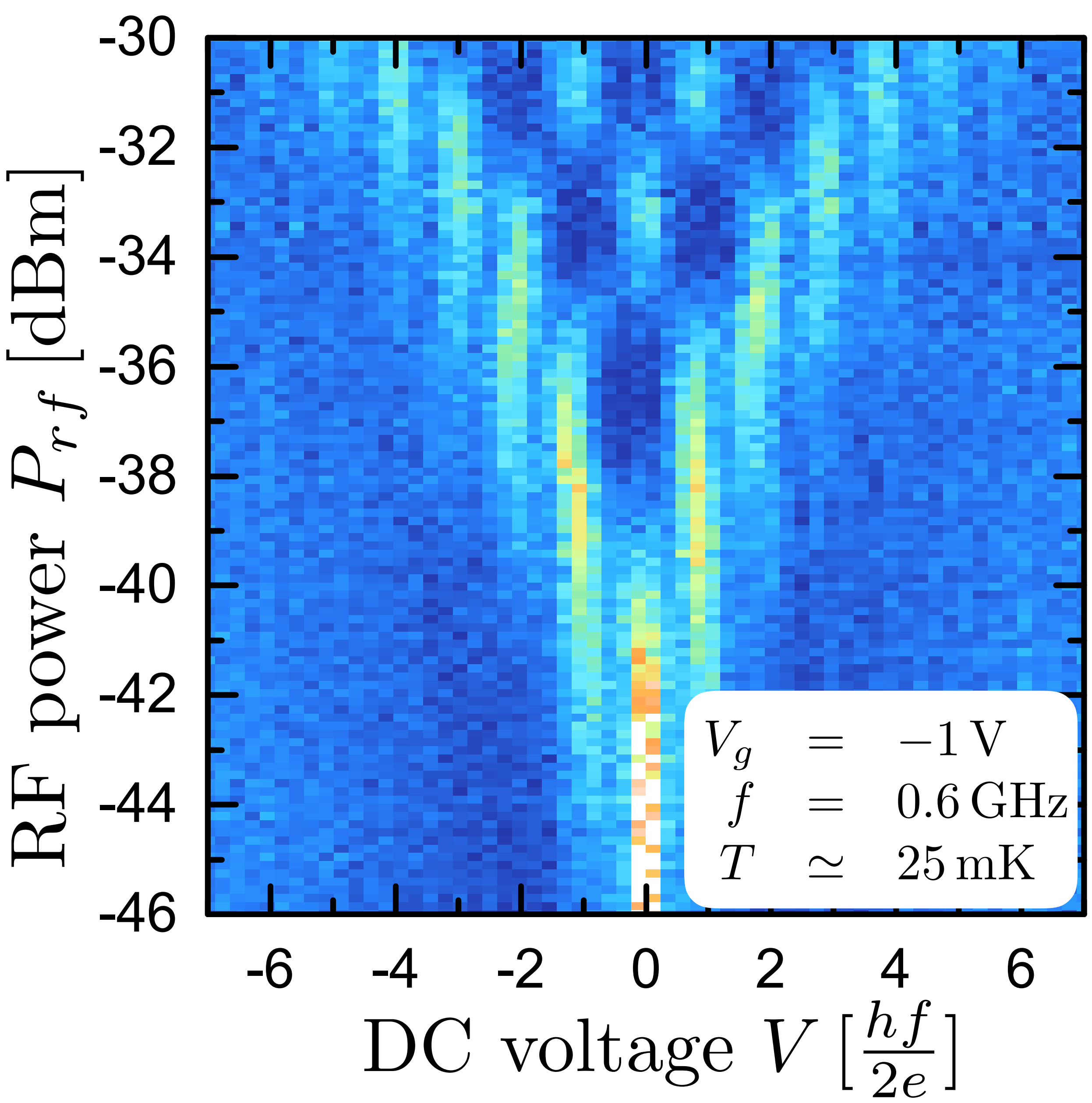
Non-topological HgTe quantum well

Non-topological QW

- narrow well (5 nm)
- no band inversion
- similar mobility 1.5 10^5 $cm^2V^{-1}s^{-1}$

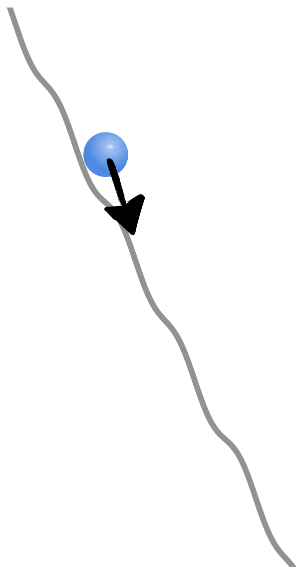
Shapiro steps

- no missing steps
- *n*-, *p*- regimes and gap verified
- down to f= 0.6 GHz



RSJ frequency dependence

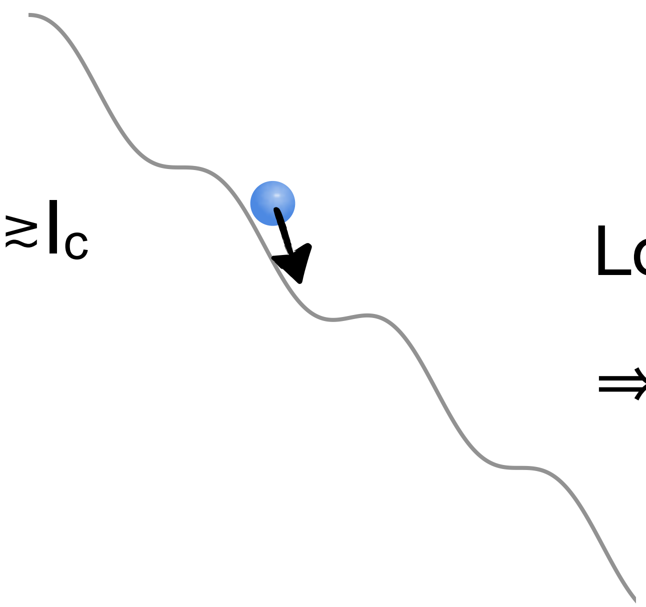
$I \gg I_c$



High frequency

⇒ almost sinusoidal

$I \gtrsim I_c$



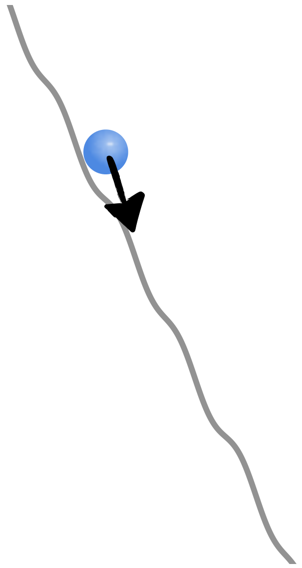
Low frequency

⇒ non-sinusoidal

Domínguez *et al.*, PRB **86**, 146503 (2012)
Domínguez *et al.*, PRB **95**, 195430 (2017)

RSJ frequency dependence

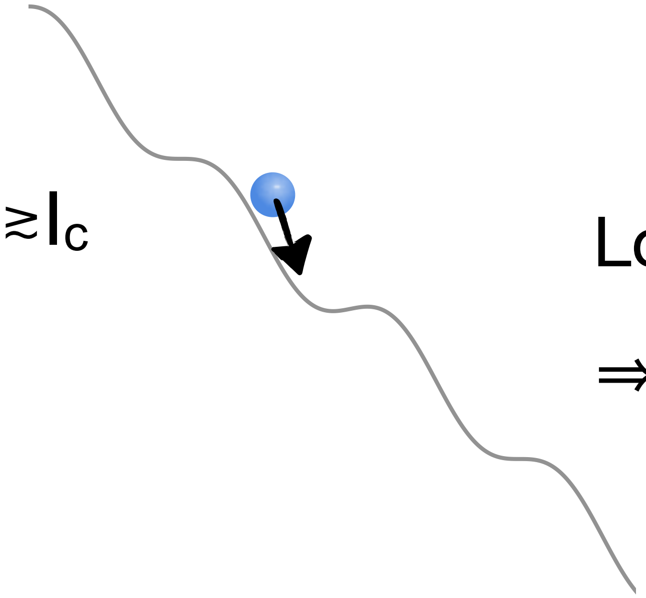
$I \gg I_c$



High frequency

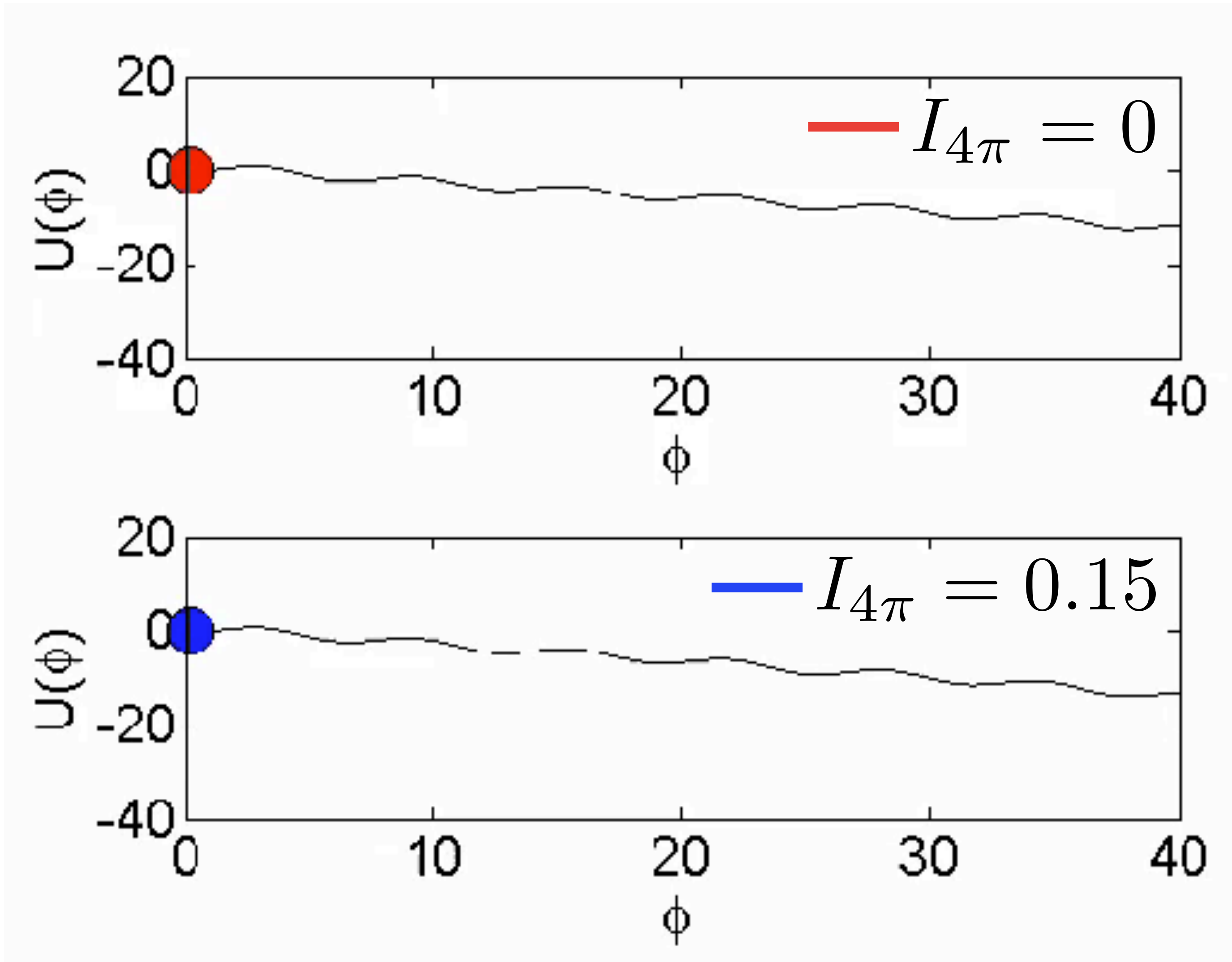
⇒ almost sinusoidal

$I \gtrsim I_c$



Low frequency

⇒ non-sinusoidal



Domínguez *et al.*, PRB **86**, 146503 (2012)
Domínguez *et al.*, PRB **95**, 195430 (2017)

RSJ frequency dependence

$I \gg I_c$

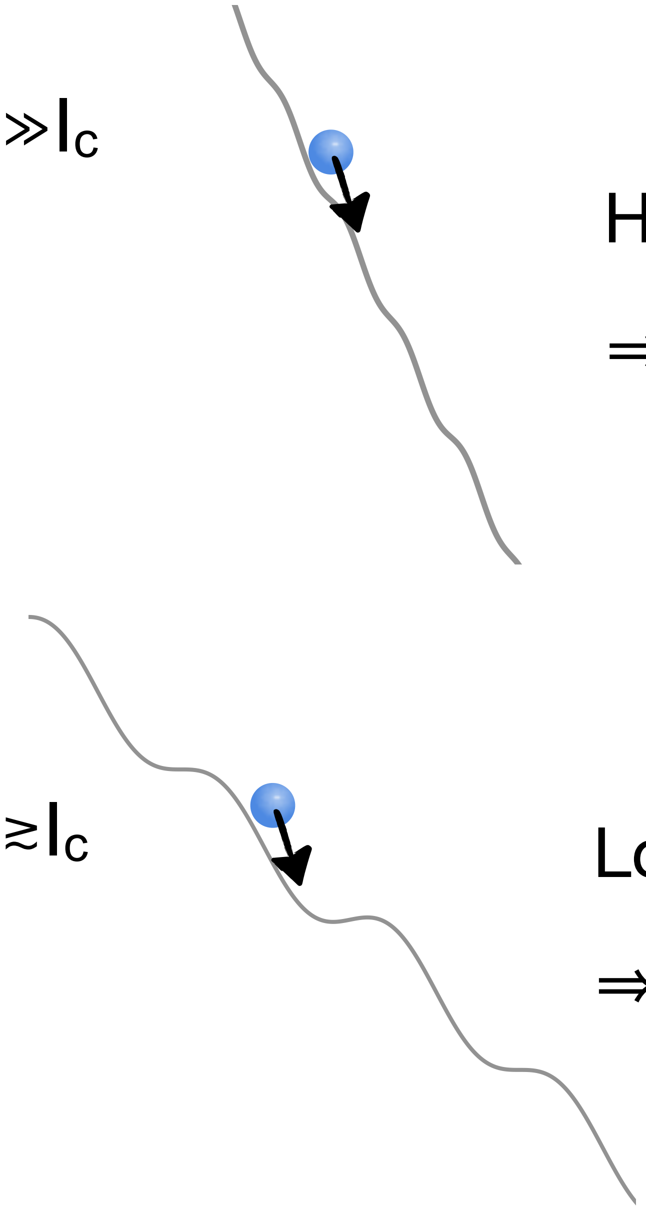
High frequency

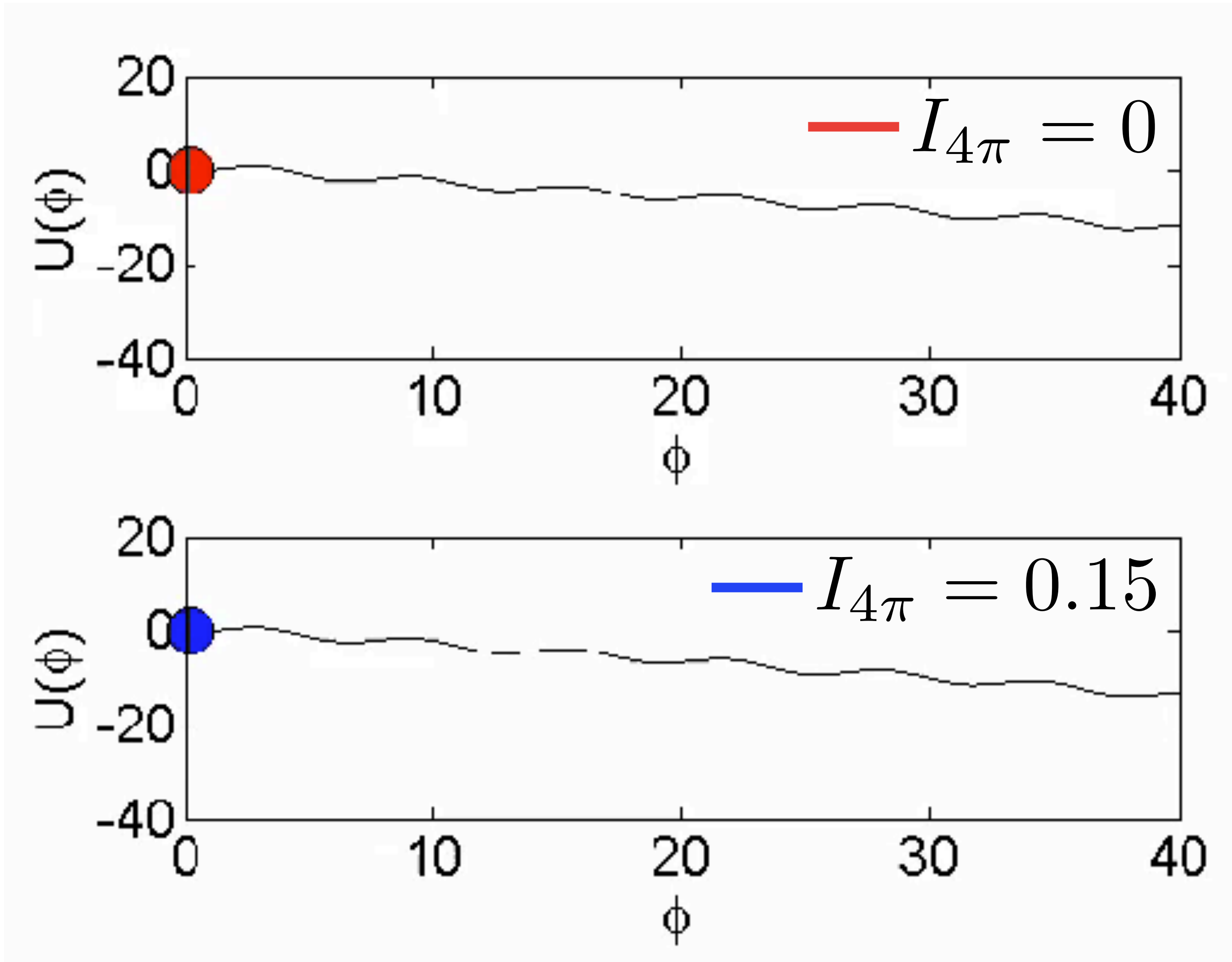
⇒ almost sinusoidal

$I \gtrsim I_c$

Low frequency

⇒ non-sinusoidal



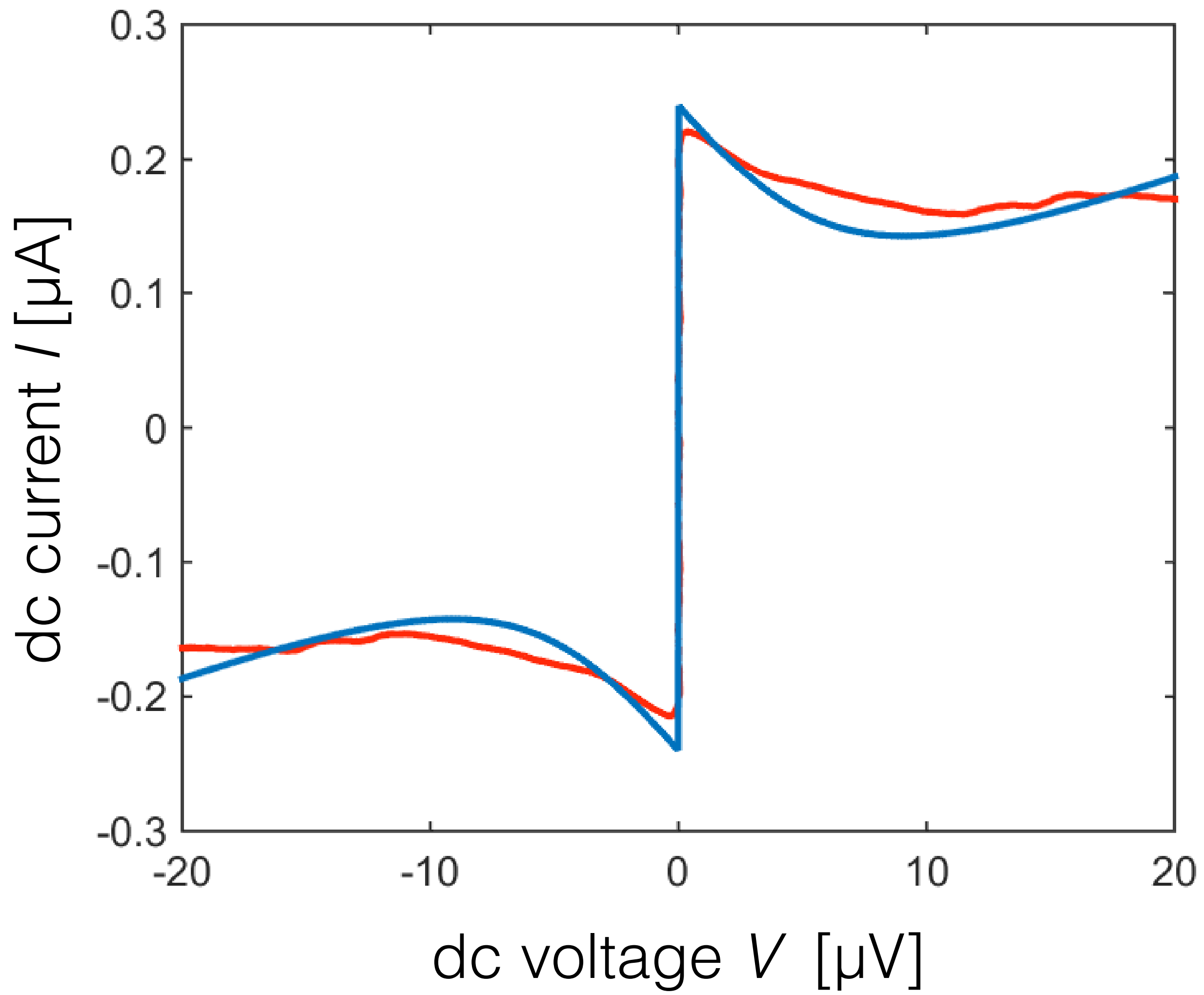


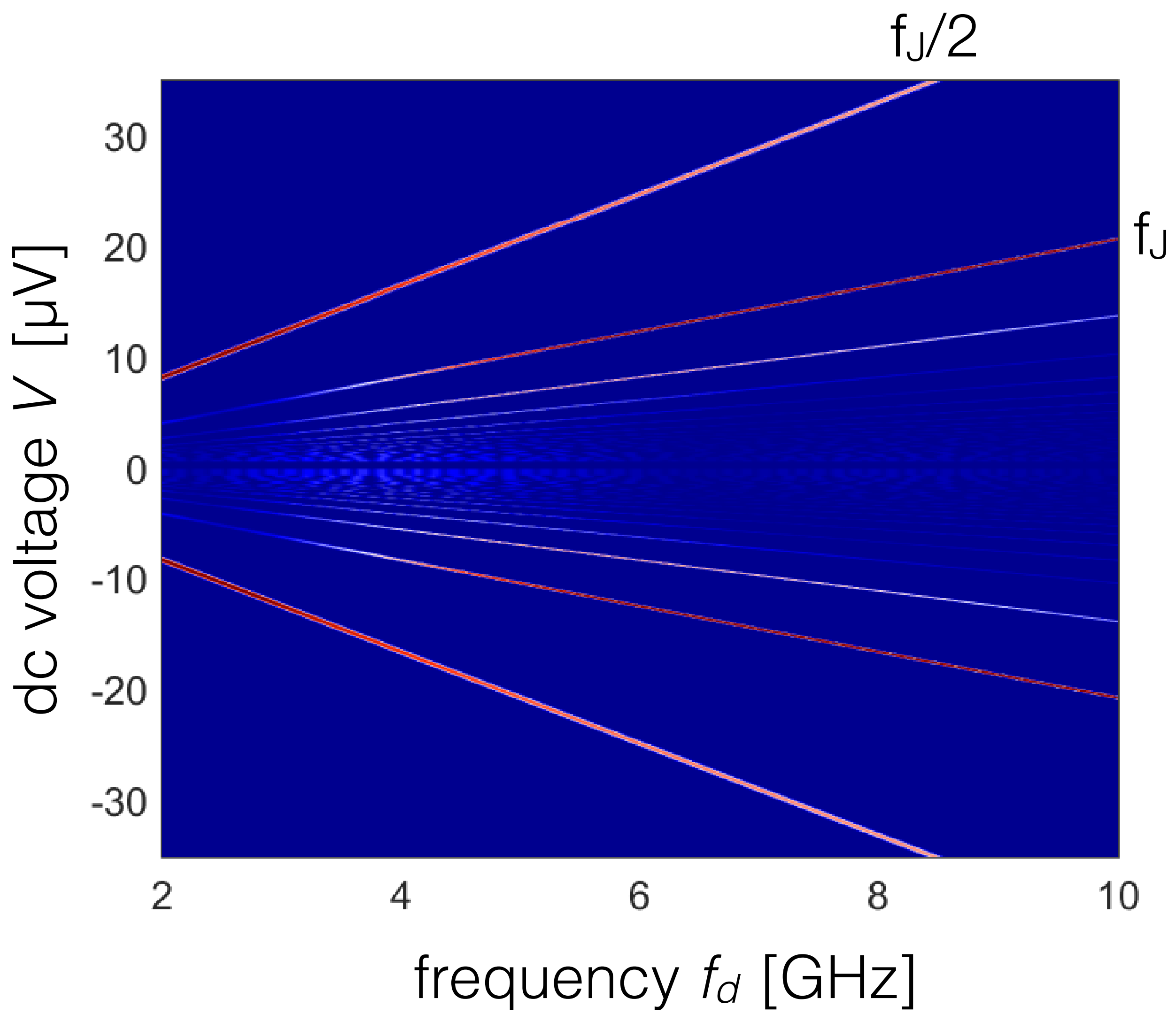
⇒ crossover $f_{4\pi}$ yields : 1-3 modes

⇒ no Landau-Zener transitions ?

Domínguez *et al.*, PRB **86**, 146503 (2012)
Domínguez *et al.*, PRB **95**, 195430 (2017)

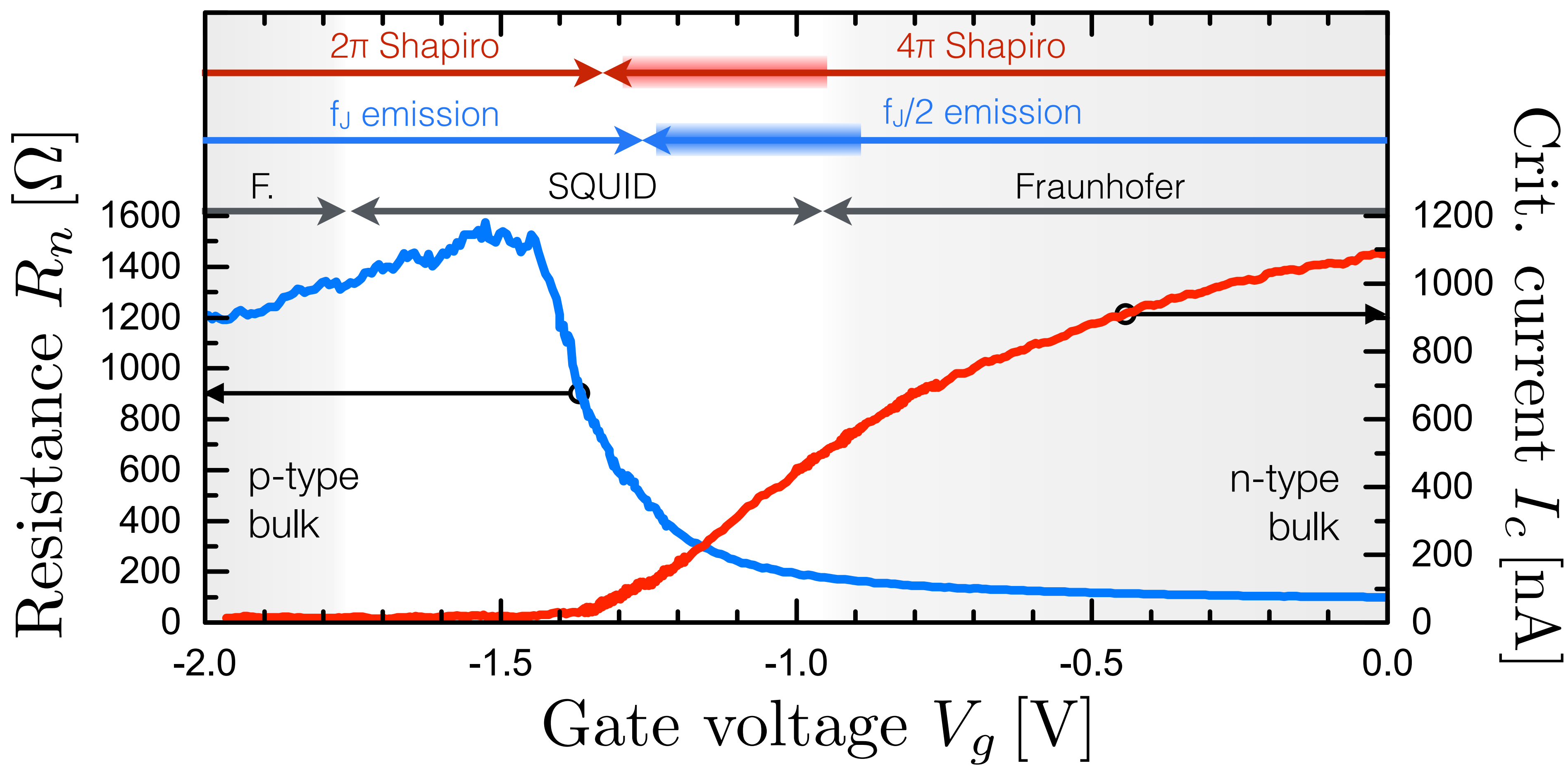
RSJ simulations





Theory : F. Domínguez & E. M. Hankiewicz

Summary of Part II



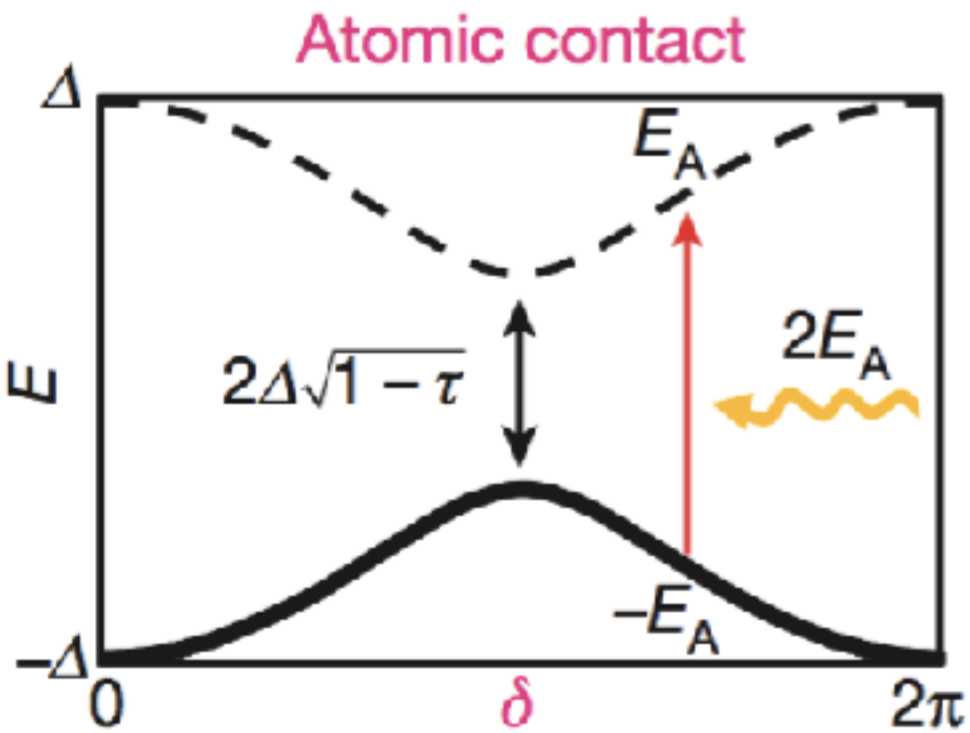
Fractional Josephson effect …

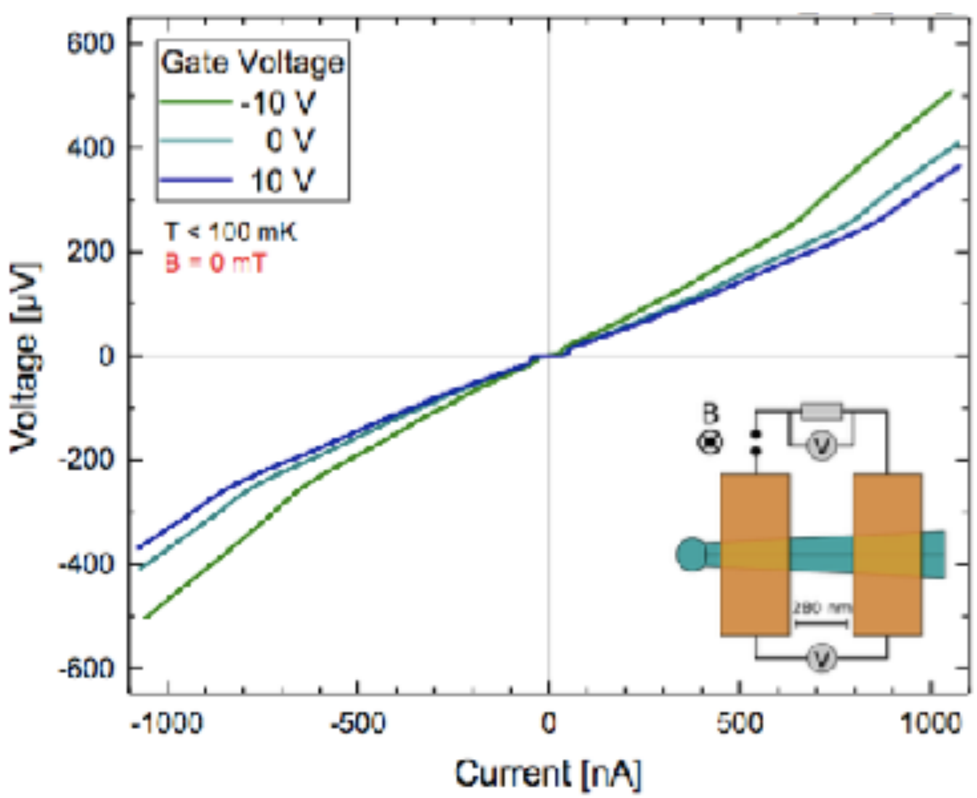
- even sequence of Shapiro steps
- emission at $f_J/2$

… of topological states ?

- Landau-Zener transitions unlikely
- edge currents (SQUID pattern)
- contribution: 1-3 modes
- coexistence with conduction band ?
- discrepancy with R_n ?

Outlook





Spectroscopy of Andreev bound states

- tunneling DOS
- absorption spectroscopy
- SN junctions

⇒ towards Majorana qu-bits

Pillet *et al.*, Nature Phys. **6**, 965 (2010)
Bretheau *et al.*, Nature **499**, 312 (2013)
Astafiev *et al.*, Science **327** 840 (2010)
Peng *et al.*, arXiv 1604.04287 (2016)

Other HgTe systems

- HgTe nanowires
- QSH, QH, QAH, Weyl

Open positions !

Thank you for your attention !

Open positions!

Post-docs

PhD students